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**BOND IMMUNIZATION:
BEYOND THE THEORY**

by
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Approved by:



A project

submitted in partial fulfillment of the requirements

for the degree of Master of Business Administration

to the Faculty of Business Administration & Economics

at Haigazian University

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Beirut, Lebanon
June 2005

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BOND PORTFOLIO IMMUNIZATION:
BEYOND THE THEORY

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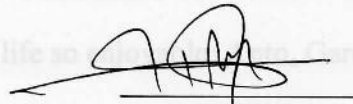
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ABSTRACT OF THE THESIS OF

Hagop Antranik Missakian for Master of Business Administration & Economics
Major: Business Administration

Entitled: Bond Immunization: Beyond the Theory.

Several authors in financial investments have investigated the validity of immunization in theory and in applied work. These authors assume different immunized portfolios, apply a few changes in interest rates, and evaluate the process by examining the results.

In this study, we examine different portfolios and define the scope of bond diversification and the selection process of the bonds in order to have more immunized outcomes. Actually, we find that forming immunized portfolios from composites of only two bonds simplifies the selection process and serves in maximizing the overall return of the investment.

We also rely on different websites to gather information on various bonds currently trading in the market. Furthermore, we form a number of immunized portfolios using different pairs of these bonds.

Because the path of interest rates is unknown, we simulate every possible path that interest rates may have for a five years time horizon by using a program in Microsoft Excel, which also calculates the end results of each portfolio in relation to each path of interest rates. Because the range of the outcomes is quite large, we use different statistical summaries and tests in order to evaluate the overall outcome of the portfolios and eventually to validate the theory of immunization.

We use histograms to plot the outcomes of each portfolio and realize that not all portfolios' outcomes have normal distributions as their corresponding data are scattered away from the mean thus increasing the risk of not being able to meet the obligation.

The analysis shows that although all the selected portfolios have means greater than the desired outcome, these portfolios do not have similar risks nor do they have the same probabilities in scoring satisfactory results and we find a relation between the risk and the realized yield of the portfolios.

We conclude by saying that immunization is workable and our analysis provides enough proof to be able to validate this theory especially that we do not use rebalancing in our study, which is one of the problems of the theory, yet we still have positive results.

| | |
|---|------|
| ACKNOWLEDGEMENTS | v |
| CONTENTS | viii |
| LIST OF FIGURES | xi |
| INTRODUCTION | 1 |
| CHAPTER I | 3 |
| A. Purpose | 3 |
| B. The Selection Process of the Bonds | 18 |
| C. Immunized Portfolio | 20 |
| D. Conclusion of the Theory | 20 |
| CHAPTER II | 24 |
| A. Duration | 24 |
| B. Immunization in Practice | 25 |
| C. The Neutralization of the Risks | 26 |
| D. Assumptions and Limitations | 27 |
| CHAPTER III | 30 |
| THE BOND PORTFOLIO SELECTION | 31 |
| A. The Single-Bond Strategy | 34 |
| B. The Two-Bond Strategy | 41 |
| C. The Multiple-Bond Strategy | 44 |
| D. Choosing the Best Strategy | 52 |
| E. Fortifying the Best Strategy | 53 |
| CHAPTER IV | 57 |
| TESTING IMMUNIZATION | 57 |
| A. The Selection of the Bonds | 57 |
| B. The Paths of Interest Rates | 61 |
| C. The Special Microsoft Excel Program | 62 |
| D. The Applied Discount Rate | 64 |
| E. The Nature of the Data | 69 |
| F. The Results of the Portfolios and the Analysis | 70 |
| CHAPTER V | 83 |
| RECOMMENDATIONS | 83 |
| CONCLUSION | 85 |
| BIBLIOGRAPHY | 87 |
| APPENDIX | 88 |

CONTENTS

| | |
|---|------|
| ACKNOWLEDGEMENTS | v |
| ABSTRACT | vi |
| CONTENTS | viii |
| LIST OF TABLES | ix |
| LIST OF FIGURES | xi |
| INTRODUCTION | 1 |
| CHAPTER 1 | 8 |
| <i>LITERATURE REVIEW</i> | 8 |
| A. Duration | 8 |
| B. Immunization | 9 |
| C. The Selection Process of the Bonds | 18 |
| D. Rebalancing the Immunized Portfolio | 20 |
| E. Contradicting the Theory | 20 |
| <i>DATA AND METHODOLOGY</i> | 22 |
| CHAPTER II | 24 |
| <i>THE THEORY</i> | 24 |
| A. Duration | 25 |
| B. Immunization in Theory | 26 |
| C. Immunization in Practice | 27 |
| D. The Neutralization of the Risks | 30 |
| E. Assumptions and Limitations | 31 |
| CHAPTER III | 34 |
| <i>THE BOND PORTFOLIO SELECTION</i> | 34 |
| A. The Single-Bond Strategy | 34 |
| B. The Two-Bond Strategy | 41 |
| C. The Multiple-Bond Strategy | 44 |
| D. Choosing the Best Strategy | 52 |
| E. Fortifying the Best Strategy | 53 |
| CHAPTER IV | 57 |
| <i>TESTING IMMUNIZATION</i> | 57 |
| A. The Selection of the Bonds | 57 |
| B. The Paths of Interest Rates | 61 |
| C. The Special Microsoft Excel Program | 62 |
| D. The Applied Discount Rate | 64 |
| E. The Nature of the Data | 69 |
| F. The Results of the Portfolios and the Analysis | 70 |
| CHAPTER V | 83 |
| <i>RECOMMENDATIONS</i> | 83 |
| <i>CONCLUSION</i> | 85 |
| BIBLIOGRAPHY | 87 |
| APPENDIX | 88 |

LIST OF TABLES

| | |
|---|----|
| TABLE I.1 | 5 |
| An Example of the Paths of Interest Rates | 5 |
| TABLE 3.1 | 38 |
| Zero-coupon Bonds' YTM According to their Term-to-maturity | 38 |
| TABLE 3.2 | 50 |
| The Possible Two-bond Combinations of the Immunized Portfolio with Time Horizon of 2.913 | 50 |
| TABLE 3.3 | 51 |
| The Sensitivity Analysis of the Bond Proportions as Calculated by Solver | 51 |
| TABLE 4.1 | 58 |
| Corporate Coupon-bearing Bonds Listed on www.investinginbonds.com as of May 5, 2005 | 58 |
| TABLE 4.2 | 59 |
| Corporate Coupon-bearing Bonds Listed on www.investinginbonds.com as of May 5, 2005 with Adjustments in the Maturities and Durations | 59 |
| TABLE 4.3 | 60 |
| All Possible Combinations for 3 years Time Horizon | 60 |
| TABLE 4.4 | 60 |
| All Possible Combinations for 5 years Time Horizon | 60 |
| TABLE 4.5 | 62 |
| Number of Possible Paths According to the Number of Years | 62 |
| TABLE 4.6 | 68 |
| The Weighted Average Realized Yield of the Selected Portfolios | 68 |
| TABLE 4.7 | 72 |
| Statistical Summaries of the Selected Portfolios | 72 |

| | | |
|------------|--|----|
| TABLE 4.8 | ANOVA of the Means of the Outcomes Using Excel | 73 |
| TABLE 4.9 | Test of Skewness and Kurtosis of the Distribution of the Outcomes | 76 |
| TABLE 4.10 | The Correlation Matrix | 78 |
| TABLE 4.11 | The Actual Standard Deviation of the Portfolios Vis-à-vis the Regressed Standard Deviation | 79 |
| TABLE 4.12 | Probability Table for the Outcomes of Portfolio G&A | 81 |
| FIGURE 3.5 | The Solutions that Solver Calculated for the Maximization Problem | 48 |
| FIGURE 3.6 | Linear Programming Sensitivity Analysis Using Solver | 51 |
| FIGURE 4.1 | The Histogram of Portfolio G&A | 80 |

LIST OF FIGURES

| | |
|--|----|
| FIGURE 2.1 | 31 |
| The Shape of the Yield Curve | 31 |
| FIGURE 3.1 | 38 |
| The Yield Curve of Corporate Coupon-bearing Bonds in the U.S as of May 2005 | 38 |
| FIGURE 3.2 | 39 |
| Comparison Between the Yield Curves of Zero-coupon Bonds and US Corporate Coupon- bearing Bonds | 39 |
| FIGURE 3.3 | 46 |
| Maximizing the YTM of the Immunized Portfolio Using Microsoft Excel | 46 |
| FIGURE 3.4 | 47 |
| Solving the Linear Programming Problem Using Solver in Excel | 47 |
| FIGURE 3.5 | 48 |
| The Solutions that Solver Calculated for the Maximization Problem | 48 |
| FIGURE 3.6 | 51 |
| Linear Programming Sensitivity Analysis Using Solver | 51 |
| FIGURE 4.1 | 80 |
| The Histogram of Portfolio G&A | 80 |

INTRODUCTION

B. Immunization

A. Interest Rates Prediction

Interest rate prediction has always been a great concern for bond portfolio managers. Actually, many studies have been conducted around the subject with the purpose of defining a certain formula or a methodology that may predict future interest rates. Yet, the forecast of interest rates, using simple forms or developed models, remains “an inexact science if not an art form”¹ and no system has ever been able to perfectly project actual figures.

Due to the above fact, information in the financial market is imperfect as the market is exposed to unanticipated changes in interest rates. Bond portfolio managers, at all times, face the risk of losing in any investment that may be subject to unfavorable movements in interest rates.

The interest rate volatility is a major point of concern also for companies and organizations that face liabilities. Pension funds and likewise financial institutions that are constrained to certain deadlines for their debts need to secure current investments in order to meet future promised payments. Both, the need for survival of such institutions and the inability to project coming periods’ interest rates, have resulted in the development of such techniques and strategies that provide protection to bond portfolios against movements in the interest rates.

C. Prior Literature

¹ Khoury (1983, p. 325) included it in chapters on “fixed-income securities” or “bond portfolio

B. Immunization

One of the most successful strategies used for managing the effects of the interest rates is called immunization, and it is based on the concept of bond duration developed by Frederick Macaulay (1938).

Over the past few decades, many authors have discussed bond portfolio immunization and its application under a category known as “bond portfolio strategies”. However, the origin of the word “immunization”, not in the biological context yet in its financial context, goes back to 1952, when Frank M. Redington, an actuary of the Prudential Assurance Company Ltd., defined “immunization” as “the investment of assets in such a way that existing business is immune to a general change in the rate of interest”.

Bond immunization strategies were developed mainly to serve banks, saving institutions, pension funds and, originally, insurance companies. The main point is that such companies and organizations carry large amounts of liabilities at any given time and hence need to guarantee their investments in order not to default on any future payment. It is worth mentioning that these liability payments can be large enough that a small change in interest rates could significantly harm the companies’ investments.

Theoretically, immunization offsets the effect of interest rate fluctuations on the invested assets to fund such liabilities as mentioned above, although, like for any other theory, assumptions and limitations exist and need to be taken into consideration when testing and applying the theory.

C. Prior Literature

Most of the available investment textbooks have explained the process of immunization and included it in chapters on “fixed-income securities” or “bond portfolio

management”, and many of the authors went into the application part of the theory and demonstrated, quantitatively, that immunization is valid.

Khoury (1983) took into consideration several cases where interest rates go up or down by an amount of 1% at different reinvestment periods. By looking at the answers in different cases, the author demonstrated that immunization is workable.

Reilly (1985) applied classical immunization by taking the example of a 10-year bond and considered a single drop of 2% in interest rates at year 5. After proving that the process of immunization guaranteed the desired wealth position of the investor, Reilly discussed another view of the theory by taking a closer look to what happens to the wealth position as interest rates change.

Later, Sharpe, Alexander, and Bailey (1999, hereafter SAB) took the example of an immunized portfolio, for two years, made up of two bonds and assumed that interest rates would either remain the same or change by 1% either way at year 1 and remain stable till the end of year 2. Through this example, the authors showed that immunization is a theory that can be applied successfully.

Similarly, Bodie, Kane, and Marcus (2002, hereafter BKM) considered an immunized portfolio made up of 6-year bonds and exposed it once to a constant rate and then to a change of 1% up and down right after the initial investment. This way, the writers showed that the terminal value of the portfolio would not be affected by the price risk and the reinvestment risk as the portfolio becomes immunized.

Obviously, these applications of immunization are based either on a single change in the interest rate with a constant rate for the remaining time period of the investment, or, dealing with a short time horizon that does not allow further movements in interest rates. Noticeably, Khoury takes the case when interest rates change twice in starting periods but a

flat rate dominates thereafter. Yet, in real life, investors experience daily changes in interest rates.

D. The Approach Used in this Study

Taking into consideration the constant change in interest rates, this study explores how strong the theory of immunization holds when immunized investments face such unstable rates and still have to, by theory, guarantee future values symmetric to what a flat rate would produce.

Interest rates can take one of three courses every period i.e. either to move upward, or downward, or remain constant. Accordingly, it is possible to have three different portfolio outcomes each referring to a path that interest rates may take.

Actually, this study is a further continuation on what SAB have started and proven to be working for a single period. Moreover, the main discussion is based on the fact that what seems to be working for a single period may be frustrating as paths multiply from as small as three, for a single period, to hundreds of paths when time horizons aim for longer periods.

The basis for this study is to take into consideration all the paths that interest rates may take through the years and apply every path on different immunized portfolios. This said, we have to note that we are not interested in every value that interest rates may have but every direction (increase, decrease, or remain constant) that they may take.

E. Paths of Interest Rates

In order to clarify what is meant by path, let us take a look at the following table.

Table I.1

An Example of the Paths of Interest Rates

| | Year 0 | Year 1 | Year 2 |
|---------|--------|--------|--------|
| Path #1 | 10% | 9% | 8% |
| Path #2 | 10% | 9% | 9% |
| Path #3 | 10% | 9% | 10% |
| Path #4 | 10% | 10% | 9% |
| Path #5 | 10% | 10% | 10% |
| Path #6 | 10% | 10% | 11% |
| Path #7 | 10% | 11% | 10% |
| Path #8 | 10% | 11% | 11% |
| Path #9 | 10% | 11% | 12% |

The first column in the table represents current interest rates and therefore rates do not change as we go from one path to another. The second and third columns represent the rates after one and two years, respectively, from the investment date. It is noted that the rates in the second and the third columns are either the same as prior years' rates or vary both ways by 1%. So, throughout this dissertation, paths represent different and independent series of interest rates whereby the relation between any two consecutive rate in any set is either 0% or $\pm X\%$ (x being any number that reflects a possible change in interest rates). In the above table $X = 1$.

F. Special Computer Program

Taking into consideration that the number of paths can become quite large and in order to simplify the testing process of this study, all of the calculations and quantitative testing of the theory are done with the aid of a particular program developed in Microsoft Excel serving mainly to summarize the answers obtained and to run the statistics for testing the validity of immunization. The details and application of the program as well as the validity testing of immunization altogether form chapter four of this dissertation.

G. The Type of Immunized Portfolios

A remarkable fact is that immunization can be applied to portfolios made up of composites of a single, double, or more bonds. However, each type of portfolio has its pros and cons and the process of building up a portfolio has a significant effect on the result of immunization. The authors mentioned earlier have randomly selected portfolios and applied immunization regardless of how many bonds are present in a portfolio. Yet, chapter two in this dissertation solely discusses the different types of immunized portfolios and poses and answers the question of which type of immunized portfolios give best results for investors?

H. Counter-assessment to the Theory

Even though immunization is taught at the MBA level in investment courses, many authors have argued that the theory has major weaknesses and have encouraged for the development of truly effective models that can serve the purpose of management. In view of that, Stewart Coutts (1993) announced that “Immunization is dead” and tried to develop an alternative model that works with cash flows and replaces the malfunctioning theory.

I. The Purpose of this Study

Actually, this dissertation is meant to be a step forward in validating the process of bond portfolio immunization and to find common grounds between two contradictory points of view. This study serves in recognizing that immunization is an effective method in securing investments, and that it is not a word that we encounter in investment textbooks.

Bond duration is one of the most fruitful and successful concepts developed in the 20th century in the field of financial investments by Frederick Macaulay (1938), and hence it is referred to as Macaulay's duration. In fact, duration can be calculated in alternative ways and that is by using different rates in discounting the cash flows. For example, Fisher and Weil (1971) used the forward rate in order to discount the cash flows of a bond, unlike Macaulay who, for the same purpose, used the yield-to-maturity (YTM) of the bond being evaluated. Bierwag and Kaufman (1977) made a study in order to identify the most appropriate and favorable concept among the alternative definitions of duration. From this study, the authors realized that the duration values were the same in all cases, and that Macaulay's definition using the YTM stands out as a primary choice, compared to the other formula of duration that needed forecasting of forward rates or making assumptions about the reasons that may have caused the change in interest rates.

In addition to the above, and besides being a measure of the average maturity of the stream of payments associated with a bond, the duration concept has played a major role as well in the development of a bond portfolio management technique referred to as immunization: the interest rate risk neutralizer of bond portfolios.

CHAPTER 1

LITERATURE REVIEW

A. Duration

Bond duration is one of the most fruitful and successful concepts developed in the 20th century in the field of financial investments by Frederick Macaulay (1938), and hence it is referred to as Macaulay's duration. In fact, duration can be calculated in alternative ways and that is by using different rates in discounting the cash flows. For example, Fisher and Weil (1971) used the forward rate in order to discount the cash flows of a bond, unlike Macaulay who, for the same purpose, used the yield-to-maturity (YTM) of the bond being evaluated. Bierwag and Kaufman (1977) made a study in order to identify the most appropriate and favorable concept among the alternative definitions of duration. From this study, the authors realized that the duration values were the same in all cases, and that Macaulay's definition using the YTM stands out as a primary choice, compared to the other formula of duration that needed forecasting of forward rates or making assumptions about the reasons that may have caused the change in interest rates.

In addition to the above, and besides being a measure of the average maturity of the stream of payments associated with a bond, the duration concept has played a major role as well in the development of a bond portfolio management technique referred to as immunization: the interest rate risk neutralizer of bond portfolios.

Factors that Affect Duration

Because immunization is directly related to duration, it is important to take a look at the factors that have an effect on duration and the way they affect it.

BKM have presented two such factors that have direct impact on duration and may create problems in the immunization process. BKM stated that there is no direct relationship between time and duration as the latter decreases at a slower pace than time. On the other hand, any change in the YTM shows its effect on duration and unbalances the latter compared to the remaining time horizon of the investment. Obviously, the impact from the above factors on duration seems to be inevitable. The passage of time and the change in interest rates cannot be avoided. Hence, at any specific period, it is almost always possible to recognize a difference between the remaining time of the investment and the duration of the portfolio.

B. Immunization

The first definition of immunization by Frank Redington (1952) involved matching assets with liabilities. Initially, the purpose of immunization was to adjust the asset portfolio in a way to match it with the duration of liabilities and hence make these portfolios mature at due dates. This strategy is more likely a “matching strategy”.

A later version of immunization, however, set the fact of having bond portfolios shielded from the risk associated with fluctuations in interest rates. In other words, immunization is the process of having portfolios of bonds that at the end of their holding periods score a certain realized yield as if interest rates remained flat throughout the years.

a. The Realized Yield

Seldom will the realized yield on an investment be equal to the portfolios' YTM as interest rates follow an unstable pattern. In essence, Fisher and Weil (1971) compared the yields to maturity with the realized returns on bonds for the period 1925-1968 and concluded that the significant difference between the two confirms the need for immunization.

Actually, the realized return on a bond is a composite of the YTM of the bond and the reinvestment rate of its coupons. Guilford and Babcock (1975) formulated an equation for the realized yield and expressed it in terms of the YTM and the reinvestment rate as follows:

$$RY = \left(\frac{d}{H} \right) K_d + \left(1 - \frac{d}{H} \right) RR \quad (1.1)$$

where, RY is the realized yield, d is the duration, K_d is the YTM on the bond, H is the holding period, and RR is the average reinvestment rate.

As an application to the above equation, consider an 8% bond with three years to maturity and a 10% YTM. This bond will have duration of 2.78 years. Let us assume that the bond is sold after two years and had an average reinvestment rate of 7% for its coupons (in fact the coupons were reinvested once at year 1 at 7%).

Therefore:

$$RY = \left(\frac{2.78}{2} \right) 10\% + \left(1 - \frac{2.78}{2} \right) 7\%$$

$$RY = 13.9\% - 2.73\%$$

$$RY = 11.17\%$$

The above answer entails that if the bond is held for two years it will have a realized yield of 11.17 %.

McEnally (1980) interpreted the above equation by stating that the realized yield is a weighted average of the YTM and the reinvestment rate. Furthermore, McEnally realized that in order to equalize the realized yield and the YTM, we need to match the duration with the holding period and, eventually, drop the risk factor associated with the reinvestment rate.

Fisher and Weil also proved that matching the duration with the desired investment horizon will immunize the portfolio. By neutralizing the reinvestment rate risk ordinary bonds will gain a characteristic common to zero-coupon bonds. Because zero-coupon bonds do not hold coupon payments, eventually, these bonds are not exposed to the reinvestment rate risk of the coupons. If held to maturity, zero-coupon bonds pay returns previously known on the date of purchase. In fact, Khoury (1983) defined immunization as the process of converting the investment portfolio into a pure discount bond. Because it pays no coupon and trades at a price below its face value, a zero-coupon bond gives its investor the exact figure of the rate of return as both, the current price and the face value, are known, and the difference between the two is the interest earned.

b. Neutralizing the Risk of the Portfolios

The importance of immunization is reflected in the fact that it works as an anti-risk or a risk neutralizer for investments against their exposure to interest rate fluctuations. Bodie, Kane, and Marcus (2002) argued that this interest rate risk can be viewed from two different angles depending on the nature of the business or the firm. BKM mentioned that bank like institutions are more interested in securing the market value of their assets, which have long-term maturities, against interest rate movements, whereas pension funds would rather secure the future value of their investments in order to meet future obligations. In both cases, the firms will be exposed to interest rate risk. However, immunization helps

those companies to insulate their financial investments against the movements in interest rates.

In fact, the unstable nature of interest rates reflects two types of risks on bond portfolios: the price risk of bonds and the reinvestment rate risk of the periodic coupons. Noticeably, Reilly (1985) started his discussion of immunization by first pointing out these two components of interest rate risk. Reilly explained that the reinvestment rate risk of the coupons cannot be avoided as all coupons will certainly not be invested and reinvested at the same rate. On the other hand, the price risk would take effect if rates change before the target period, and the bond is sold prior to its maturity. Because immunization can be applied on diversified portfolios, it is important to be aware of the price risk in this case, as investors may have to precipitate the selling of considerable amounts of bonds at unexpected prices before maturity.

Khoury (1983) elaborated the counter-effect of the reinvestment rate risk and the price risk on one another; as interest rates increase, coupons will be invested at higher rates. Yet, due to the inverse relation between the price and the yield of a bond, the increase in the yield will decrease the price hence acquiring capital losses. On the other hand, the decrease in the yield will result in a lower income from the reinvested coupons and a higher bond price. Khoury explains how at the duration, the mounting up of the losses from reinvesting the coupons, as rates have decreased, is totally compensated by the price increase and vice versa. When the portfolio is immunized, reinvestment rate risk effect and the price risk effect totally offset each other.

c. Assumptions and Limitations

Nevertheless, the theory of immunization is based on several assumptions and limitations. One of the biggest assumptions of this theory, as put by Fisher and Weil

(1971), is to consider that the change in interest rates is common to all rates and with a similar amount for all. In other words, they have assumed that the yield curve is flat and the shifts in the curve are parallel. Justifying this assumption, Fisher and Weil demonstrated that immunization is possible only if the duration is equal to the desired investment period.

Immunization, however, does not rely on a single assumption. Sharpe, Alexander, and Bailey (1999) listed a few problems that an investor may face with immunization. For example, if a bond in an immunized portfolio defaults any payment at any time during its holding period, the effects will be great on immunization. Also, when a bond is called by its issuer, immunization will not be complete. Actually, duration and immunization are based on the assumption that bonds will not default and will not be called. In order to overcome this problem, one can choose before hand non-callable bonds or treasury securities. Yet, in case when bonds include different options, effective duration helps as an alternative measure for duration. Bonds may be callable, puttable, or defaulting. Yet, effective duration is a measure that takes into consideration the embedded options in bonds and offsets the effect of exercising such provisions.

One of the assumptions that SAB have mentioned among the problems of immunization was the same as the one that Fisher and Weil identified to be the non-parallel shifts in the yield curve. In addition, SAB stated that a shift in the yield curve may have a greater effect on short-term securities more than it is on long-term securities. Hence, in a diversified portfolio, bonds may be affected differently due to non-parallel shifts in the yield curve. These shifts would cancel out immunization. Alternatively, SAB introduced the concept of cash matching that remains unaffected by non-parallel shifts in the yield curve. Matching the cash inflow with the promised outflow at a certain date is also referred to as a dedicated portfolio.

Another factor that contributes to the occurrence of problems is the fact that an investor is open to a variety of candidate portfolios that have durations of the required time span. SAB have represented a solution to this problem and that is to choose a portfolio that has the highest average YTM. Also, choosing bonds with duration closer to the duration of the cash outflows is a preferable choice for the portfolio.

d. Immunization in Practice

Verifying the validity of immunization needs the involvement of quantitative analysis and a few applications on the theory. A shallow application would be to consider a portfolio of bonds with duration equal to the investment's time horizon, then to apply a few changes in interest rates, and to compare the end wealth of the immunized portfolio with the desired wealth position. In fact, in chapter 4 of this dissertation we will be verifying the validity of immunization using more advanced and realistic approaches and we will be applying the theory on several portfolios. Yet, we have to note that a variety of quantitative applications have been actually carried out to test the power that the theory holds.

Khoury (1983) considered a variety of cases when interest rates change differently. In fact, he took the example of a pension fund needing a certain amount of money after 3.29 years from the current date. Knowing the limited amount in hand the pension fund needed to invest at a rate that guaranteed the desired end wealth. For his example, Khoury stated that in order to fulfill the desired outcome the whole amount should have been invested at a certain calculated flat rate. Yet, rates are never flat and immunization remains the solution.

Khoury (1983), in his example, assumed that there were only two available investments in the market and these were bond A and bond B. Both bonds paid 12% semi-annual coupons. Also, the duration of bond A coincided with the term-to-maturity of bond

B that was equal to the investor's time horizon of 3.29 years. Noticeably, bond A satisfied the duration criteria and was suitable for immunization whereas bond B was not. Khoury calculated the total cash in hand after 3.29 years of exclusively investing in bonds A or B. In fact, he considered six different bond market events and assumed a different scenario for each. Also, he assumed that rates would change by 1% either way and on different dates ending up by the following six cases: rates would rise or fall by 1% either immediately or after 7 months, and rates would rise or fall twice consecutively by 1% after 3 and 7 months from the date of purchase. By this example, Khoury demonstrated that immunization is applicable and also workable.

In each case, Khoury summed up the income received from the coupons and their reinvestments with either the selling price of the bond after 3.29 years in the case of bond A, or the principal value in the case of bond B. He then compared the realized yields of bond A and bond B in each scenario and came to the conclusion that only for a single immediate shock in interest rates, bond A would have had a realized yield equal to the required rate of return. As for the other scenarios, the calculated yields for bond A were not significantly different from the desired yield whereas the gap between the yield of bond B and the desired yield was realizable in each and every scenario.

Similarly, Frank Reilly presented an example of classical immunization. Reilly compared the maturity strategy with the duration strategy by using a single change in interest rates. For this purpose, he used an 8% eight-year bond and another 8% ten-year bond. The latter had duration equal to 8.12 and not exactly 8. The only event that took place during the eight years holding period was assumed to be a single 2% drop in rates prior to the payment of year 5.

After running the calculations, obviously, the results of the duration strategy were satisfactory. In fact, the decrease in the rates resulted in a shortfall in the reinvestment cash

flow from year 5 and up. The less income from the reinvestment process was supposed to be offset by the price appreciation of the bond. However, the maturity strategy has eliminated the price risk. On the other hand, the duration strategy had an ending wealth much closer to the desired outcome as the effects of the decrease in the reinvestment rate were offset by the increase in the price of the ten-year bond sold 2 years prior to its maturity.

On the other hand, an increase in interest rates would allow the maturity strategy to have an outcome higher than the desired wealth. Notably in this case, the duration strategy will have an ending wealth less than it is for the maturity strategy. However, the main point is that, although the maturity strategy can sometimes be preferable and more rewarding, the duration strategy has less variation and eliminates the risk by scoring realized wealth positions identical to what was expected. This means that, when we apply several fluctuations in interest rates, the duration portfolio will still score a desirable outcome whereas the outcome of the maturity strategy would vary unfavorably.

Bodie, Kane, and Marcus (2002) have only focused on the immunized portfolio without taking into consideration the maturity matched portfolio. Interestingly, BKM provided a real life situation that insurance companies face. They have considered the example of an insurance company that issues a guaranteed investment contract (GIC) for an individual's retirement-saving account. In fact, GICs are zero-coupon bonds issued by insurance companies to their customers.

After a GIC is issued, the insurance company will need to invest in a certain portfolio to guarantee the payment at the maturity date of the GIC.

BKM considered that a \$10,000 GIC is issued and needs to be paid after five years given that the due amount is the future value of the GIC at the market rate of 8%. BKM supposed that the company decided to invest \$10,000 in 8% coupon bonds selling at par

with a term-to-maturity of six. The portfolio will include ten of such bonds and will have duration of five.

BKM took the case where rates may fall by 1% reaching 7% or rise to 9% right after the investment is made. The results in both cases showed harmony with the desired outcome and proved that, as the duration of the portfolio equaled the term to maturity of the GIC, the portfolio was fully immunized and also showed a surplus in the ending position.

year 2. SAB had a totally different approach on how immunization is accomplished for bond portfolios. SAB considered a portfolio made up of two different bonds with different durations and characteristics. Actually, they have taken into consideration one-year 7% coupon bonds and three-year 8% bonds. So, the one-year and three-year bonds have durations of 1 and 2.78 respectively given that the YTM is 10%. In order to build an immunized portfolio made up of these two bonds we need to know which portions of these bonds satisfy our purpose. For this reason, SAB provided two equations with two unknowns. The first equation entailed that the summation of the shares of the bonds has to be equal to 1. The second was to fulfill the criteria of duration and this involved making the weighted average of the durations equal to the term of the investment.

composition Taking the example of a firm that needs to arrange a million dollars to pay an obligation after two years, SAB showed how immunization can be applied on that. From the equations, the portion of each bond in the portfolio was calculated. Now in order to define the amount of investment in each bond, the calculated portions were multiplied by the present value of the needed sum. Then, the investment amounts were divided by the price of each bond thus they were translated into the number of bonds needed for the investment. Actually, all these details can be done by simple mathematical procedures. As for testing immunization, SAB considered that rates may remain the same, increase by 1%,

or decrease by this same amount at the end of one year yet remain constant for the second year.

Notably, at maturity, the one-year bond has to be reinvested for another year. In order to calculate the aggregate value of the portfolio, SAB added up four components: value at year 2 from reinvesting one-year bonds proceeds, value at year 2 from reinvesting the three-year bonds' coupons of year 1, value of three-year bonds' coupons received at year 2, and the selling price of the three-year bonds at year 2. The results proved that the needed one million dollars was fully funded in all three cases. Actually, the Excel program that we will use in chapter four will calculate the end results of different portfolios by using the same approach as SAB.

In summary, whatever bonds are selected to form the immunized portfolio, interest rates fluctuations will no more have an impact on the desired end position or final wealth.

C. The Selection Process of the Bonds

Khoury (1983) mentioned an interesting concept concerning the selection process of the bonds for the immunized portfolio. Actually, he recommended not to include composites of only a single bond in the portfolio and provided the required justifications in a few points.

First, there may not be enough issues of that single bond in order for an investor to put the whole amount in hand in that particular security. Second, usually managers like to diversify thus reducing the unsystematic risk of their portfolios and that is by investing in more than a single bond. Third, including more than one bond in the portfolio makes the process of rebalancing more flexible. To illustrate this last point of his, Khoury stated that by the passage of time, the duration of the portfolio cannot be adjusted if it only includes a

single bond. On the other hand, adjusting the proportions of the various bonds in a diversified portfolio will surely do its purpose.

As for selecting the bonds, Khoury pointed out two criteria. (1) Selecting bonds with maturities equal or above the time horizon. (2) Selecting the portfolio that maximizes the YTM.

To illustrate the selection procedure of the bonds, Khoury took a numerical example and considered five different Treasury notes from an issue of the *Wall Street Journal*. Knowing the YTM of each note and after calculating their durations, he formulated a problem that required the solution of linear programming.

Actually, the purpose of linear programming was to assign a weight or a proportion to each note in a way to form a portfolio with the highest possible YTM. The problem was to maximize the weighted average YTM of the portfolio subject to two constraints: the first one was used to set the weighted average maturity of the bonds' durations equal to the time horizon of the investor, and the second one was assigned to equate the summation of the proportions to be 1, and finally each proportion was set to be greater than or equal to zero.

With the use of linear programming, the proportion of each note in the optimal portfolio was calculated. Only two out of five weights were greater than zero as the remaining proportions did not enter the solution.

From this realistic approach, Khoury gave a clear idea of a method that managers might use in real life to build their immunized portfolios.

Taking into consideration the fact of having assumptions and limitations in the theory of immunization, Stewart Coutts (1993) focused on points that weaken the overall theory and transform it into a redundant application. For example, Coutts mentioned that

² At 10% discount rate the perpetuities' duration is $1.10 / 0.10 = 11$ years.

D. Rebalancing the Immunized Portfolio

Immunization needs constant rebalancing or else, as Khoury (1983) puts it, after some passage of time the bonds included in the portfolio will no longer be suitable in serving their purpose.

An interesting numerical procedure was adopted by BKM and aimed to provide another real life example concerning the rebalancing process of the immunized portfolio.

For the algebraic demonstration, BKM used a combination of 3-year pure discount bonds with perpetuities paying coupons. Because the current interest rate was set to be 10%, the perpetuity will have duration of 11 years². Supposing the time horizon to be 7 years, BKM calculated that the two investments will have equal proportions of 0.5 and total dollar investments of \$5,000 each (the whole amount being \$10,000 that is the present value of the obligation to be paid in seven years). As one year passed by, the time horizon went down to six. As a result, the initial proportions did not satisfy the equation anymore. So, new proportions were set to be 5/9 for the zero-coupon bonds and 4/9 for the perpetuities. Yet, the funds have risen from ten to eleven thousand dollars. 5/9 of this new dollar value is \$6,111.11 to be invested in the 3-year bonds. Actually, the zero-coupon bonds have increased 10% in value to become \$5,500 and thus needing the \$500 coupon of the perpetuity plus the selling of \$111.11 of the latter and the investing of the proceeds in the zero-coupon bonds.

E. Contradicting the Theory

Taking into consideration the fact of having assumptions and limitations in the theory of immunization, Stewart Coutts (1993) focused on points that weaken the overall theory and transform it into a redundant application. For example, Coutts mentioned that

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not only profits are immunized but also losses. Also, he argued that the theory does not include derivatives, futures, and options that may serve to protect the equity funds. Coutts also stated that immunization does not take into account the uncertainty of liabilities, the timing of the payments, and the alternative distributions of the assets. Alternatively, Coutts suggested the involvement of computer programs to run simulations, which we will use in our study, and hence taking into consideration the different factors that affect the immunization process.

Finally, Coutts tried to develop a model that may work on computers given a structured algorithm. He included nine steps in the model and claimed that immunization is one particular case of the model.

The last part of this dissertation includes taking into consideration various immunized portfolios. These portfolios will be formed by different bonds in the United States (US) market whereby the information is derived from the website investinginbonds.com that lists a variety of corporate bonds. All the data from all the sources are related to the US bond market.

In order to simulate the paths of interest rates, we will use a special program developed in Microsoft Excel that will also provide a wide range of results and statistical

DATA AND METHODODLOGY

The theoretical parts of this project are sourced from various investment textbooks of the past few decades. These textbooks include *Investments* by Bodie, Kane, and, Marcus (2002), *Investments* by Sharpe, Alexander, and Bailey (1999), *Investment Analysis and Portfolio Management* by Frank Reilly (1985), and *Investment Management: Theory and Application* by Khoury (1983).

This dissertation aims at verifying the validity of a theory in financial investments called “immunization”. For this, various financial bond markets related data need to be contributed in the analytical parts of the chapters. Mainly, the data is sourced from trustworthy and reliable sources on the internet. The data employed in this study come from websites like Bloomberg.com, investinginbonds.com, and federalreserve.org. Chapter four deals with the selection process of the bonds and thus the data and the charts in the chapter are from Bloomberg.com. The purpose of these data is to compare the yield curves of zero-coupon bonds and that of coupon-bearing bonds. Also, chapter two includes data on treasury notes from an old issue of the *Wall Street Journal* that are directly sourced from Sarkis Khoury (1983).

The last part of this dissertation includes taking into consideration various immunized portfolios. These portfolios will be formed by different bonds in the United States (US) market whereby the information is derived from the website investinginbonds.com that lists a variety of corporate bonds. All the data from all the sources are related to the US bond market.

In order to simulate the paths of interest rates, we will use a special program developed in Microsoft Excel that will also provide a wide range of results and statistical

summaries that aim at verifying the validity of immunization. In fact, every path will have a different effect on the portfolio and eventually will result in a different outcome. The chain of outcomes will be the data that we will study. According to these data, the Excel program will provide statistical summaries ranging from descriptive statistics to hypothesis testing. In short, these summaries will be sufficient to validate or not the theory.

development and the usage of functional strategies that neutralize interest rate effects are viewed as rewarding.

One of the most popular and commonly practiced techniques is *Immunization*. Immunization enables investors to protect their investments from fluctuations in interest rates applying easy to use methods and simple procedures.

Immunization is the process of investing in securities in a way that makes the portfolio unaffected from fluctuations in interest rates and allows it to have an end position similar to the case where interest rates would have had remained constant throughout the holding period of the investment.

The purpose of this chapter is to pave the way to better understand the idea behind immunization and to form a clear picture of the whole theory. Moreover, the definitions and the explanations referred to throughout the chapter will form the base of this dissertation and will serve as a reference to the remaining parts.

As mentioned above, immunization does not involve complexities especially in the application part whereby only a few criteria need to be employed.

As a matter of fact, the whole theory of immunization is based on the concept of duration that was developed in 1938 by Frederick Macaulay. At this point, it is important to mention that throughout the chapters, the word "duration" will refer to Macaulay's duration and not to another definition that may have a dissimilar usage.

CHAPTER 2

THE THEORY

It is difficult, if not impossible, to forecast interest rates. For this reason, the development and the usage of functional strategies that neutralize interest rate effects are viewed as rewarding.

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A. Duration

As a definition, duration is a measure of the average maturity of the streams of payments associated with fixed-income securities³. Subsequently, duration serves as an accurate alternative measure of the maturity of bonds and deals with the ambiguity of the latter⁴. Notably, the terms “duration” and “effective maturity” are used interchangeably as both have the same meaning.

In fact, calculating the duration of a bond involves a few steps. The first step includes the discounting of each promised future cash flow to its present value (PV) and the multiplying of each by its corresponding year number i.e. the yearly time interval after which the cash inflow will be obtained. Next, all the proceeds of the previous calculations need to be summed up then divided by the market price of the security. In a mathematical format, duration is expressed as follows⁵.

$$D = \frac{\sum_{t=1}^T PV(C_t) \times t}{P_0}, \quad (2.1)$$

where $PV(C_t)$ represents the present value of the cash flow at year t , and P_0 represents the market price of the bond at the current year.

In the case of a zero-coupon bond the duration is always equal to its term-to-maturity as the price of such a bond is equal to the present value of the cash flows. Hence, in the above equation the present value and the price are simplified, as both are equal, leaving only $D = t$.

³ Sharpe, Alexander, and Bailey (1999 p. 424)

⁴ Bodie, Kane, and Marcus (2002. p.485-486)

⁵ Sharpe, Alexander, and Bailey (1999 p. 425)

Besides the duration of the bond, there is the modified duration that is a figure measuring the effect of the changes in interest rates on the price of the bond. The modified duration is expressed as:

$$D_m = \frac{D}{(1 + y)} \quad (2.2)$$

Where: y is the interest rate

Actually, whenever there is a change in interest rates, the price of the bond will change by $\Delta y \times D_m$ which will be expressed in percentage change in the price of the bond.

B. Immunization in Theory

Immunization is the process of building up bond (or other security) portfolios in a way to make them unaffected by movements in interest rates. More specifically, immunization is a “reasonable risk management policy”⁶ that allows managers to shield their investments against changes in interest rates thus guaranteeing future positions equivalent to what they would have been if rates were flat. Because interest rates are never constant, adopting a strategy such as immunization is favorable to investors as it allows some level of certainty to the ending wealth position of their portfolios.

Most commonly, immunization is a technique used by many financial institutions that are subject to certain future obligations. The purpose is to be certain that the investments will provide the necessary amounts when debts are due.

⁶ Bodie, Kane and Marcus (2002. p. 501)

C. Immunization in Practice

Although its function is quite amazing, the way immunization is accomplished is very simple. Basically, immunization is based on a single criterion which is to set up the portfolio in a way that makes its duration equal to the time horizon of the investor. The latter is a value displayed in years and it represents the time interval between today and the due date of the payment invested for. In other words, the time horizon is the term-to-maturity of the obligation.

This said, companies and/or investors needing a certain amount of money at a specified date in the future can invest in a bond, or a portfolio of bonds, having duration equal to the time interval between today and the liability date and consequently immunize their investments. In nature, immunization works for a previously given set period of time hence it is important to first, identify a future obligation and then to construct the immunized portfolio based on the time horizon of the identified debt.

As an example, consider that a company has to pay a certain amount of money after three years from the current date. When the company decides to immunize a portfolio in order to fund the obligation after three years, it will be enough to construct a portfolio where the weighted average duration is three, no matter how many bonds are included in the portfolio.

Actually, matching the duration of the portfolio with the time horizon of the investor is the main procedure in the whole process of immunization. Another important procedure would be the selection of the bonds that will form the immunized portfolio. For this, several options exist. Immunized portfolios can be made up of composites of one, two, or more bonds as long as the overall duration of the portfolio matches the time horizon. However, these different types of portfolios have different impacts on immunization. The selection process of the bonds will be thoroughly discussed in chapter three. Here it is

sufficient to note that every decision can have its pros and cons on the immunized portfolio and, consequently, only one decision has to be identified as the best choice.

In the case where a single bond is selected for the portfolio, the latter's duration will be equal to that of the selected bond. In case two or more bonds are selected for the process, the overall duration of the portfolio will be the weighted average of all the individual bonds' durations present in that portfolio. In an algebraic format this can be viewed as: $\sum X_i d_i$, where X is the proportion of each bond in the portfolio and d its corresponding duration. Yet, the weighted average duration has to match the time horizon of the investor therefore we set $\sum X_i d_i = T$, where T is the time horizon, and we also set $\sum X_i = 1$. Unlike the arbitrage portfolio, the immunized portfolio cannot have a bond with a negative proportion thus one needs to make sure that not only all proportions add up to 1 but to also have positive proportions. With the aid of all these equations, it is possible to calculate the proportion of each bond.

Actually, these calculated proportions serve in knowing how much of the total needed investment has to be assigned to which bond in the portfolio. The needed amount of investment is nothing but the present value of the future obligation. Hence, each proportion is multiplied by the total investment amount to obtain the dollar value of the investment in that particular bond, and then the answer is divided by the price of the bonds to know the total number of issues of that bond that needs to be bought. So eventually, the proportions are translated into the number of bonds to be bought. Finally, buying the calculated amounts of the different bonds sets forth the immunized portfolio. Of course, in case an investor decides to buy issues of only a single bond, it will be enough to divide the whole amount needed by the price of that bond to obtain its needed number of issues.

Actually, when we determine the proportions of the bonds in the portfolio and then calculate the number of issues of these bonds to buy, the number of bonds may include fractions. However, it is not needed to round these numbers because we assume that it is possible to buy fractions of these bonds as long as we know its dollar investment figure. This is called the divisibility of bonds.

Although immunization can be considered as a passive management strategy, managers are indeed active during the whole holding period of the investment with a few exceptions. These exceptions include the buy-and-hold strategy of composites of single zero-coupon bond issues.

When coupon-bearing bonds are involved in the immunized portfolio, the received coupons have to be all invested and reinvested year after year at current rates till the maturity of the obligation.

Now, in case more than two bonds are present in the immunized portfolio, management needs to be active in different ways. In reality, the fact that some bonds will mature prior to the maturity of the obligation has given the “active” characteristic for immunization. In a diversified immunized portfolio, a certain amount of bonds will be maturing prior to the date of the obligation and the bond holder will get paid the principal values of the bonds plus the last coupon payment of each. In this sense, bonds may mature a year or more, prior to the due date. The proceeds of the maturing bonds will need to be reinvested for the remaining period of years in the time horizon. The reinvestment process of these proceeds is done at a yearly basis i.e. to reinvest the proceeds year after year at the current reinvestment rate till the obligation becomes due.

At the moment, we have covered the theoretical procedure of the whole process of immunization. Yet, the bond selection procedure and the scope of diversification of the

immunized portfolios will be our focus in chapter three as these will contribute in the obtaining of favorable outcomes.

D. The Neutralization of the Risks

At this point, many questions are raised like for instance how does immunization work? How does the interest rate increase or decrease not affect the end result of the investment? How can we explain the function of immunization? The following few pages provide answers and explanations to such questions.

Actually, the interest rate risk is composed of two different risks. The first component of risk is related to the fact that the interest rate movements affect the yields of bonds, which in their turn have a direct impact on the prices of these bonds. Therefore, changes in interest rates affect the price of a bond and create what is called the *price risk* (also referred to as interest rate risk). The second component of the interest rate risk is related to the reinvestment process of the coupons that will be received and reinvested periodically. Indeed, at every new period, the reinvestment rate will be different than its prior level due to fluctuations in interest rates hence injecting uncertainty in the reinvestment process and shaping a kind of risk referred to as the *reinvestment rate risk*. For any given change in interest rates, the two risks will behave in opposite directions. Specifically, an increase in interest rates will be in favor of the reinvestment rate risk, as coupons will be invested at higher rates, but will increase the risk on the price of the bond making its devaluation the end result of such a movement in interest rates.

Every time interest rates move, whether up or down, we may have two outcomes that is either favorable or unfavorable to investors. Thus, we will have four possible situations or scenarios and these are: (1) rates move upward favorably (2) rates move

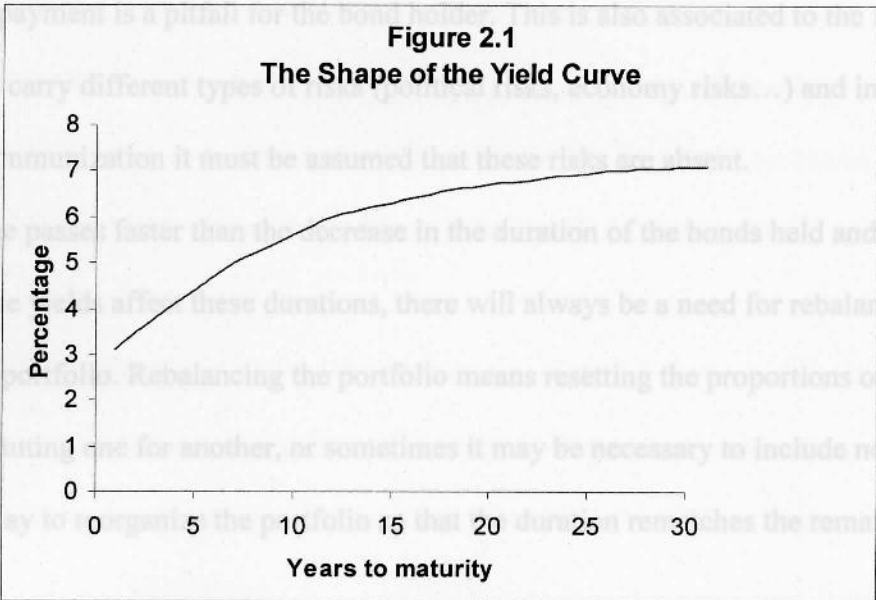
downward favorably (3) rates move upward unfavorably (4) and rates move downward unfavorably.

As a matter of fact, the secret of immunization is embedded in the power of perfectly counterbalancing the effect of the two risks. In other words, immunization has the power to make the more/less reinvestment income and the capital gains/losses counterbalance each other whatever the movements in interest rates are.

E. Assumptions and Limitations

Naturally, every theory is subject to assumptions and limitations. Actually, immunization is a theory that relies on several assumptions and may have certain limitations. In essence, investors need to be aware of the various assumptions and limitations of this theory, although sometimes they may interfere in the overall result of immunization.

The major assumption of immunization is what Fisher and Weil (1971) referred to as non-parallel shifts in the yield curve. Figure 2.1 represents the ordinary shape of the yield curve.



Obviously, the yield curve is neither a straight line nor a well defined curve where there may be a relation between the slopes at different points. From the above curve, we can conclude that rates rise as maturities become longer and that maturities increase more rapidly than yields do. Applying this on immunization we can notice that in a diversified immunized portfolio, bonds have different maturities and therefore a change in the yield curve will not affect all bonds' yields similarly. Thus, any given change in interest rates will have a different effect on different bonds given their maturities. Investors applying immunization on their investments have to assume that the yield curve is flat and that the shifts in the curve are parallel, or try to identify bonds having durations close to one another so that movements in the yield curve may similarly affect the bonds. This means that immunization assumes to have parallel shifts in the yield curve thus the YTM of the different bonds in the immunized portfolio will be affected similarly and will change by the same amount.

Another assumption, rather a problem, is the fact that bonds will not be bought back by their issuers who may exercise their rights in calling the bonds. Concurrently, it is assumed that bonds will not default any payment, whether periodic or at maturity, because defaulting any payment is a pitfall for the bond holder. This is also associated to the fact that bonds may carry different types of risks (political risks, economy risks...) and in order to accomplish immunization it must be assumed that these risks are absent.

As time passes faster than the decrease in the duration of the bonds held and also the change in the yields affect these durations, there will always be a need for rebalancing the immunized portfolio. Rebalancing the portfolio means resetting the proportions of the bonds by substituting one for another, or sometimes it may be necessary to include new bonds all in a way to reorganize the portfolio so that the duration rematches the remaining time horizon.

The fact that there is a wide range of bonds available in the bond markets translates into having endless options for forming immunized portfolios. The immunization criteria mentioned earlier in the chapter can be applied on a large number of different bond combinations. Actually, the scope of diversification of these portfolios with the selection process of the bonds will be a subject to discuss in the following chapter.

subject that has been included in the immunization process nor has it been thoroughly discussed. In the literature review section, we mentioned how each author selected different portfolios that were made up from different individual bonds. The authors, more commonly, used several cases where a single bond was present and the immunized portfolio was a composite of that same bond. However, the construction of portfolios is not only restricted to include a single bond. The possibilities are endless. At one extreme, choosing a single bond seems to be relatively an easy option; on the other, making diversified portfolios with two or more bonds may positively affect the investment. Yet, the selection from the available possibilities creates confusion among investors and therefore enhances the complexity of the immunization process.

In this section, we will examine each and every possible type of portfolio and we will then describe the selection process of the bonds for each of them. Nevertheless, our interest is mainly to relate each type of portfolio to its corresponding effect on immunization. Finally, after summing up the outcomes, we will be able to decide on the alternative that will have the most positive effect on the theory of immunization.

A. The Single-Bond Strategy

Let us first start our discussion by assuming that an investor is only interested in a single security to construct the immunized portfolio. We will define this strategy as the single-bond strategy. Based on chapter 2, we have already discussed that in order for a

CHAPTER 3

THE BOND PORTFOLIO SELECTION PROCESS

The process of selecting bonds in building up immunized portfolios is not a subject that has been included in the immunization process nor has it been thoroughly discussed. In the literature review section, we mentioned how each author selected different portfolios that were made up from different individual bonds. The authors, more commonly, used several cases where a single bond was present and the immunized portfolio was a composite of that same bond. However, the construction of portfolios is not only restricted to include a single bond. The possibilities are endless. At one extreme, choosing a single bond seems to be relatively an easy option; on the other, making diversified portfolios with two or more bonds may positively affect the investment. Yet, the selection from the available possibilities creates confusion among investors and therefore enhances the complexity of the immunization process.

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A. The Single-Bond Strategy

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portfolio to be immunized, its duration must be equal to the time horizon of the investor; and we took on the example of 3 years time horizon entailing that a portfolio with duration 3 is enough to achieve immunization. Building on the same example, the investor that chooses the single-bond strategy will actually be searching for bonds having duration of three. Generally, the duration of a bond is not valid information in bonds' listings. The Wall Street Journal, Bloomberg, and many other financial data and information provider sources are adequate to mention the years to maturity of the bond and consider duration as extra information that seekers have to calculate themselves.

Of course, pure discount bonds have durations equal to their years to maturity. For the time being, let us ignore zero-coupon bonds and keep our focus on coupon-bearing bonds.

This being said, a tremendous amount of time and effort need to be invested in the process of calculating hundreds of bonds' durations and in order to identify the one that will match the desired time horizon. Although the process of spotting a bond that satisfies the duration condition seems to be possible, an investor caught up with calculations may fall in a vicious trap harming the overall investment. Indeed, the concentration on computational processes of this manner distracts the consideration of other factors such as the bond's yield to maturity and/or its market price as they affect immunization in the following ways.

The existence of the relationship between a bond's price and its yield to maturity makes it worthwhile to simultaneously discuss their effect on immunization. In fact, a low yield to maturity will result in a high price and it will unfavorably impact the outcome. So, spotting the bond with the accurate duration that matches the time horizon may jeopardize the end result of the portfolio. Actually, the identified bond may be unattractive in the sense of having less YTM compared to other bonds with the same maturity, or it may even be too

risky. As investors may neglect the yield to maturity and/or the price of the bond, the outcome would result in the shrinking or the growing of the number of bonds that the investor will be able to invest in. Actually, the higher the number of bonds involved in immunization the greater the benefit. To illustrate this, let us take a look at the following example. An investor with \$100,000 cash ready to be invested in bonds is confused between choosing two available, mutually exclusive, options: bond A or bond B. The investor will use all of the funds by either purchasing bond A or bond B until the total amount is spent. We have to note that the divisibility of bonds will make the purchase of fractions of these bonds a possible action. Both bonds have one year to maturity and pay 5% coupon. Yet, bond A has a 6% yield to maturity selling at \$990.57 and bond B an 8% yield to maturity and hence selling at \$972.2 a bond. Now, with \$100,000 in hand, the investor will be able to buy 102.85 of bond A whereas with the same amount he/she can only afford to buy 100.95 of bond B. Buying less bonds translates into getting paid less coupons. In this case, if the investor chooses bond B, at the end of one year, he/she will be paid \$5,047.5 from coupons which is less by \$95 than if he/she chooses bond A. Though the difference is not big in this example, dealing with millions of dollars and much longer time horizons can significantly show the difference, as fewer coupons will be received each year. In essence, the less the received coupons are the less they have the opportunity of being invested and reinvested, period after period, till the time horizon expires, causing even much severe losses.

Concurrently, a higher yield to maturity works in favor of immunization and offers the following advantage points.

First of all, a high YTM means a higher return on the bond. Second, when the yield is high, any drop in interest rates will represent a less percentage decrease in the YTM of the bond. For instance, if two bonds have yields to maturity of 9% and 10% and there is

1% decrease in rates, there will be an 11.11%⁷ decrease in the yield of the first bond and only a 10%⁸ decrease in the yield of the second.

Alternatively, choosing zero-coupon bonds is an option that eases the process of equalizing the duration with the time horizon. The simplicity of the process comes from the fact that zero-coupon bonds have durations equal to their years to maturity. A three-year zero-coupon bond is sufficient to achieve the immunization of a portfolio for three years time horizon. Consequently, and in order to select the most rewarding zero-coupon bond, it will be enough to compare the yields to maturity of other available 3-year pure discount bonds and to choose the one with the highest yield. Actually, as Khoury stated, the purpose of immunizing a portfolio is to make it a zero-coupon bond. Yet, one has to consider the drawbacks of choosing zero-coupon bonds for immunization.

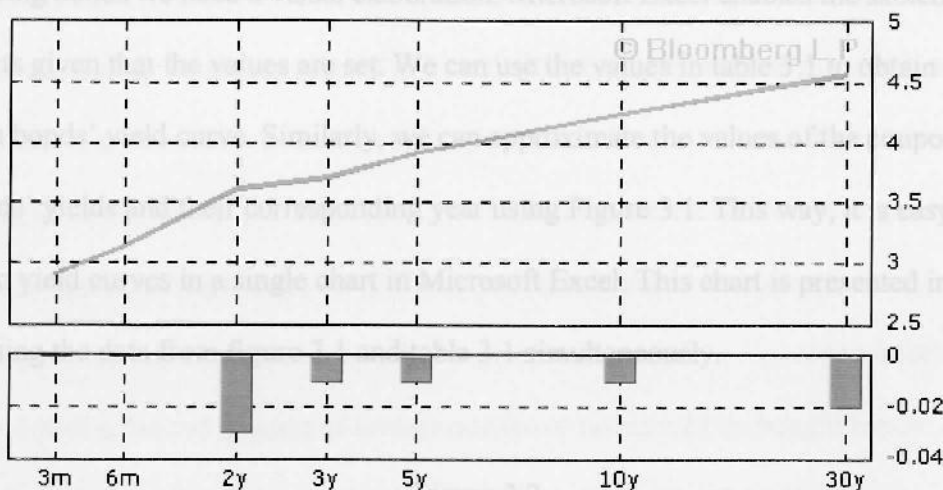
Primarily, zero-coupon bonds pay less yield-to-maturity. This information can be derived from trustworthy sources on the internet. Let us use Bloomberg.com in order to compare the yields-to-maturity of zero-coupon bonds to those that pay periodic coupons.

Figure 3.1 on the next page is obtained from bloomberg.com and it represents the yield curve of US corporate coupon-bearing bonds. The chart presented in the figure plots the years to maturity of the bonds on the X axis and its corresponding yield-to-maturity in percent on the Y axis. From the chart, many conclusions can be derived and many points can be realized. For instance, from the yield curve we can conclude that, on average, three years-to-maturity U.S. corporate coupon-bearing bonds have a yield of 3.6%. Now, let us take into consideration the zero-coupon bonds' yield curve and compare the different yields according to the bonds' maturities.

⁷ Percentage change in the YTM is equal to the amount change in the YTM (new YTM – old YTM) divided by the old YTM: $1\% / 9\% = 11.11\%$

⁸ Percentage change in the YTM is equal to the amount change in the YTM (new YTM – old YTM) divided by the old YTM: $1\% / 10\% = 10\%$

Figure 3.1
The Yield Curve of Corporate Coupon-bearing Bonds in the U.S as of May 2005



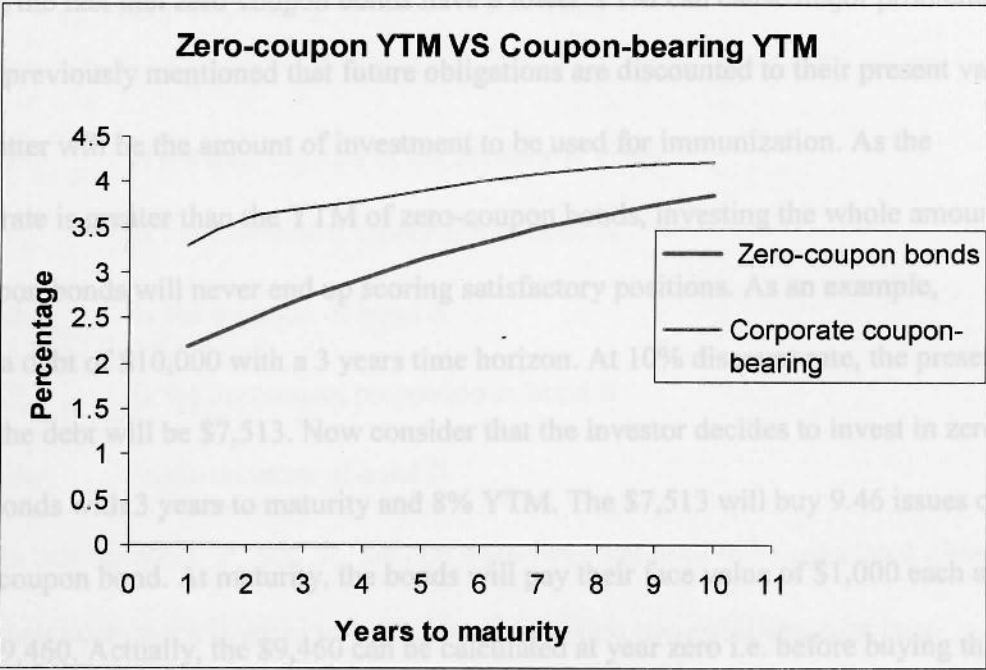
Actually, Bloomberg does not provide a chart for the yield curve only a table describing the YTM of the zero-coupon bonds according to the years-to-maturity. These data points are available on the website of Bloomberg and presented in table 3.1.

Table 3.1
Zero-coupon Bonds' YTM According to their Term-to-maturity

| Years to maturity | Zero coupon yield ri(%) | Years to maturity | Zero coupon yield ri(%) |
|-------------------|-------------------------|-------------------|-------------------------|
| 1 | 2.19 | 16 | 4.26 |
| 2 | 2.47 | 17 | 4.3 |
| 3 | 2.72 | 18 | 4.34 |
| 4 | 2.94 | 19 | 4.37 |
| 5 | 3.15 | 20 | 4.4 |
| 6 | 3.33 | 21 | 4.42 |
| 7 | 3.49 | 22 | 4.45 |
| 8 | 3.63 | 23 | 4.47 |
| 9 | 3.76 | 24 | 4.49 |
| 10 | 3.86 | 25 | 4.5 |
| 11 | 3.95 | 26 | 4.52 |
| 12 | 4.03 | 27 | 4.53 |
| 13 | 4.1 | 28 | 4.54 |
| 14 | 4.16 | 29 | 4.55 |
| 15 | 4.21 | 30 | 4.55 |

In order to be able to compare the yields of the zero-coupon bond with those of coupon-bearing bonds we need a visual elaboration. Microsoft Excel enables the sketching of such charts given that the values are set. We can use the values in table 3.1 to obtain the zero-coupon bonds' yield curve. Similarly, we can approximate the values of the coupon-bearing bonds' yields and their corresponding year using Figure 3.1. This way, it is easy to draw the two yield curves in a single chart in Microsoft Excel. This chart is presented in figure 3.2 using the data from figure 3.1 and table 3.1 simultaneously.

Figure 3.2
Comparison Between the Yield Curves of Zero-coupon Bonds and US Corporate
Coupon- bearing Bonds



By examining figure 3.2, it is obvious to realize that the gap between the two curves is decreasing as we move upward in the years to maturity. However, the blue line does not intersect with nor does it increase beyond the red line, which means that for any

given year-to-maturity a coupon paying bond will always yield higher than a zero-coupon bond will. For instance, for 5 years to maturity, zero-coupon bonds pay around 3% YTM whereas coupon-bearing bonds pay almost 4%. Actually, the differences in the yields are linked to the fact that zero-coupon bonds hold no reinvestment rate risk and their earned interest is known at the date of purchase. So, in order to compensate the reinvestment rate risk factor of coupon-bearing bonds, the YTM of such bonds has to be higher.

We have already explained how a lower yield will result in a higher price for the bond and how with a limited amount of investment fewer bonds will be bought hence smaller number of coupons will be received. Here, we cannot argue about the fewer coupons as in this case the bonds are bought at discount and they do not pay any coupon. However, the fact that zero-coupon bonds have a lower YTM can cause major problems. We have previously mentioned that future obligations are discounted to their present value and the latter will be the amount of investment to be used for immunization. As the discount rate is greater than the YTM of zero-coupon bonds, investing the whole amount in zero-coupon bonds will never end up scoring satisfactory positions. As an example, consider a debt of \$10,000 with a 3 years time horizon. At 10% discount rate, the present value of the debt will be \$7,513. Now consider that the investor decides to invest in zero-coupon bonds with 3 years to maturity and 8% YTM. The \$7,513 will buy 9.46 issues of the zero-coupon bond. At maturity, the bonds will pay their face value of \$1,000 each and a total of \$9,460. Actually, the \$9,460 can be calculated at year zero i.e. before buying the zero-coupon bonds which means that the negative gap will occur whatever the fluctuations in interest rates will be. In case an investor decides upon the single zero-coupon bond strategy there will be a willful decision in not being able to entirely fund the obligation in the future.

B. The Two-Bond Strategy

A second alternative for building immunized portfolios is to proportionally include composites of two different bonds. Actually, choosing this type of portfolios is the easiest and most effective method among the alternatives presented throughout this chapter. Let us consider this strategy to be the two-bond strategy.

As always, the most important criterion is to have the duration of the immunized portfolio equal to the time horizon. In the previous section, the duration of the selected bond represented the portfolio's duration. Now in this case, as we have two bonds and their corresponding durations, the duration of the portfolio equals the weighted average of the durations of the individual bonds. Mathematically, the latter can be written as:

$$T = P_A \times dur_A + P_B \times dur_B \quad (3.1)$$

where:

T is the time horizon of the investor in years

P_A is the investment proportion in bond A

dur_A is the duration of bond A

P_B is the investment proportion in bond B

dur_B is the duration of bond B

In this equation, P_A and P_B are two unknowns that need to be calculated given the fact that their summation is equal to 1. This way we obtain two equations with two unknowns and these are:

$$T = P_A \times dur_A + P_B \times dur_B$$

$$P_A + P_B = 1$$

In fact, an investor willing to apply the two-bond strategy for immunization can choose any two bonds from the market that he/she may find attractive. As we have seen in

the previous section, choosing a zero-coupon bond is not a favorable choice for investors in terms of YTM, hence it remains irrelevant in all cases.

In addition to the above two equations, there is another one that needs to be taken into consideration when we proceed with the calculations. In fact, we have to add that P_A and P_B are both positive and we can not accept a negative number for these proportions. The latter can also be translated into the fact that one of the bonds has to have the duration shorter than the time horizon, and the other has to have the duration longer than the latter. Otherwise, we may end up having one negative proportion and another positive one, and the proportions would hit numbers in the few hundreds percent. Let us mathematically prove the above since we cannot accept negative proportions and we will always have the time horizon in between the two durations.

$$P_A + P_B = 1 \text{ therefore we have } P_A = 1 - P_B \text{ and } P_B = 1 - P_A$$

Replacing P_A by $(1 - P_B)$ in equation $T = P_A \times dur_A + P_B \times dur_B$ we obtain:

$$T = (1 - P_B) \times dur_A + P_B \times dur_B$$

$$T = dur_A - P_B \times dur_A + P_B \times dur_B$$

$$T = P_B (dur_B - dur_A) + dur_A$$

$$P_B = \frac{T - dur_A}{dur_B - dur_A}$$

Similarly, if we replace P_B by $(1 - P_A)$ in equation 3.1 we get:

$$P_A = \frac{dur_B - T}{dur_B - dur_A}$$

So, if $dur_B > dur_A$ then the denominator of both equations will be positive.

In this case, for both proportions to be positive, we need to have both numerators to be positive i.e. $T - dur_A > 0$ hence $T > dur_A$, and $dur_B - T > 0$ hence $dur_B > T$. Adding them together we obtain: $dur_A < T < dur_B$

On the other hand, if $dur_B < dur_A$ then the denominator of both equations will be negative. Now, in order not to have a negative proportion, we have to make sure that the numerators for both equations are negative. Therefore, we need to have $T - dur_A < 0$ hence $T < dur_A$, and $dur_B - T < 0$ hence $dur_B < T$. Adding them together we obtain: $dur_A < T < dur_B$

We see that in both cases, we need to have one of the durations to be less and the other to be greater than the time horizon.

This being explained, the identifying of the two bonds for the portfolio can be divided into two separate searches: (1) searching for an attractive bond given that its duration is shorter than the time horizon. (2) Searching for an attractive bond given that its duration is longer than the time horizon. At this stage, we are still in the process of choosing an alternative, so it is better to first, verify the claim that the two-bond strategy is the best, and later to build on the findings.

From this point of view, it is noticeable how much easier it is to use the strategy of including two bonds in the immunized portfolio rather than to complicate the process and focus on a single bond. Solving two equations with two unknowns is a one-minute by hand calculation and choosing two attractive bonds is nothing but the job of investors.

Comparing with the one-bond strategy, the strategy described in this section can be seen as advantageous because it conserves time, effort, and eventually money in the process of seeking securities. Besides, other advantages that result from portfolio diversification, such as the risk factor, also provide superiority to the two-bond strategy. Actually, in the last

section of this chapter we will also be able to verify that choosing these two bonds in a way to have their durations close to the time horizon even results in better outcomes.

C. The Multiple-Bond Strategy

Another way of selecting a portfolio is to involve a higher number of bonds and further diversify the investment. In this section, we will discuss how an investor can immunize a portfolio made up of three or more bonds. Furthermore, our main focus will be to compare the results and pull out the advantages and the disadvantages of such strategy. It is worth pointing that every time we increase the number of bonds by one, we will be adding two other variables to our equations: the duration of the bond and the latter's proportion of the investment.

Such portfolios were never been tested by the authors mentioned previously in the literature review section. However, Khoury, with the use of linear programming, ended up selecting two out of five initially selected bonds as those have actually maximized the yield of the portfolio. Indeed, the discussions and the calculations in this section will allow us to verify if the “two” that Khoury ended up with is always the case or just a number by chance. Actually, since the linear programming problem that we will set has only two major constraints (aside from the non-negativity constraints), the number of basic variables will always be 2 thus only two bonds will be selected. However, we will apply the bond selection process using Microsoft Excel and eventually we will come to the same conclusion that only two bonds will be present in the optimal solution.

Let us first start by looking at what happens to our equations as we plan to add a bond to the process.

$$T = P_A \times dur_A + P_B \times dur_B + P_C \times dur_C$$

$$P_A + P_B + P_C = 1$$

This set of equations includes three proportions that need to be calculated by using the two available equations. Such calculations are not possible unless we desire to assign a number to one of the proportions and calculate the others, or we translate the given into a linear programming problem. If we choose the first option and try to apply it even when more bonds are included, it will become less professional, as we will be assigning proportions till we end up with only two unknowns. Therefore, it is preferable to use linear programming to enrich the results with precision and a great extent of accuracy.

In Microsoft Excel, there is a command named Solver that is especially designed to solve linear programming problems. We will apply several cases on Solver as it will be contributing to fulfill our purpose of defining the scope of portfolio diversification for better immunization.

Before going into the details of the program and running some applications, let us first write the problem in a linear programming format and define what needs to be maximized. Of course, maximizing the yield to maturity is our objective and we have already discussed the advantages of higher yields in the first section of this chapter, so its importance is realizable at this point. In a portfolio of three or more bonds, the yield to maturity is equal to the summation of the product of each proportion of bond and its YTM. In other words, the yield to maturity of the portfolio is the weighted average yield of all the bonds in the portfolio. Maximizing the yield to maturity of the portfolio means maximizing the weighted average.

Therefore, in a linear programming context, where Y is the YTM of the bonds, P the corresponding proportions, and where five bonds are selected, the problem can be written as follows:

$$\text{Max } P_A \times Y_A + P_B \times Y_B + P_C \times Y_C + P_D \times Y_D + P_E \times Y_E$$

Subject to:

$$P_A + P_B + P_C + P_D + P_E = 1$$

$$P_A \times dur_A + P_B \times dur_B + P_C \times dur_C + P_D \times dur_D + P_E \times dur_E = T$$

$$P_A, P_B, P_C, P_D, P_E \geq 0$$

This maximization example is based on the fact that there are only five bonds selected for the process of identifying the portfolio with maximum YTM. Yet, it is possible to include as much bonds as desired and to make the proper adjustments in the given.

We have to note that, in the case of the two-bond strategy, there was no discussion concerning the weighted average yield to maturity of the portfolio. Yet, the maximization process of the yields was part of the selection and identification of the individual bonds, as each had its own search field. A more detailed explanation is yet to come.

Figure 3.3 provides the format of a maximization problem inputted in Excel. The numbers used in the figure 3.3 are the same as Khoury had used. The purpose is to make sure that Solver is used properly.

Figure 3.3
Maximizing the YTM of the Immunized Portfolio Using Microsoft Excel

| | A | B | C | D | E | F | G | H |
|---|-------------------|--------|--------|--------|--------|--------|-----------|-------|
| 1 | | Bond A | Bond B | Bond C | Bond D | Bond E | Portfolio | Total |
| 2 | Duration | 2.881 | 3.155 | 3.486 | 3.541 | 3.738 | 3.2913 | 0 |
| 3 | Yield-to-maturity | 12.57% | 12.43% | 12.41% | 12.31% | 12.46% | 0 | |
| 4 | Proportion | 0 | 0 | 0 | 0 | 0 | 1 | 0 |

As we can see in the figure, there are some cells with zero values. Actually, these cells will automatically be filled as we run Solver and they represent the solutions we are

looking for. Cells B4 to F4 represent the proportions of the bonds to invest in. We can see in G4 the number 1 which is the summation of all the proportions. The time horizon is 3.2913 it is also the duration of the portfolio. Given the yield-to-maturity and the duration of all the bonds, it is time to maximize and find out what proportions of which bonds will result in maximum yield-to-maturity of the portfolio. But first, we have to insert a formula in G3, H2, and H4, which represent the weighted average YTM, the weighted average duration of the bonds that will be selected by the program, and the sum of the proportions that will be assigned by Solver, respectively. These formula are:

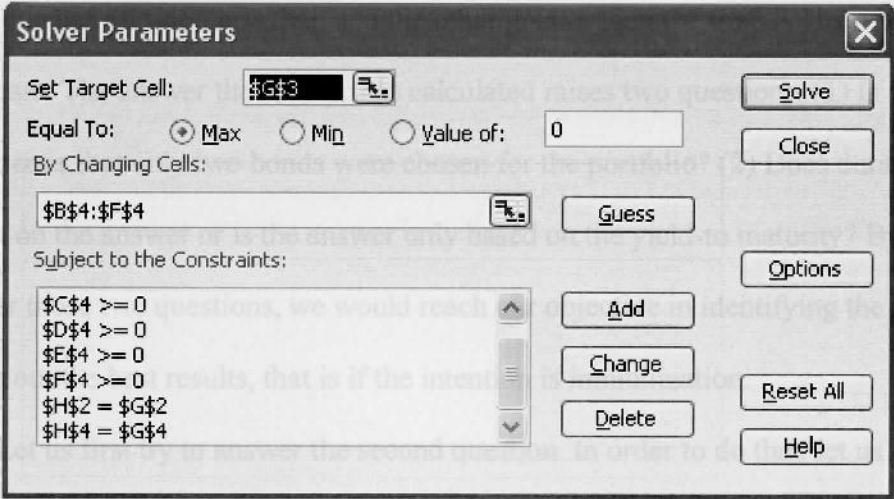
$$G3=B3*B4+C3*C4+D3*D4+E3*E4+F3*F4$$

$$H2= B2*B4+C2*C4+D2*D4+E2*E4+F2*F4$$

$$H4=B4+C4+D4+E4+F4$$

In order to input the problem in Solver we click on Tools and choose Solver. A window will appear and the input is done in the following way as can be seen in the screen shot in Figure 3.4.

Figure 3.4
Solving the Linear Programming Problem Using Solver in Excel



bond E. Yet, the answer does not change and the optimal solution is the same. Actually,

Figure 3.4 shows that the target cell is G3 which is the weighted average YTM of the portfolio that we are maximizing. This maximum YTM will be calculated by changing cells B4 to F4 where we set the proportions of the bonds to be. Finally, we add all the constraints needed for the problem and click on the button Solve to obtain the solution that is displayed in Figure 3.5.

Figure 3.5

The Solutions that Solver Calculated for the Maximization Problem

| | A | B | C | D | E | F | G | H |
|---|-------------------|---------|--------|--------|--------|----------|-----------|--------|
| 1 | | Bond A | Bond B | Bond C | Bond D | Bond E | Portfolio | Total |
| 2 | Duration | 2.881 | 3.155 | 3.486 | 3.541 | 3.738 | 3.2913 | 3.2913 |
| 3 | Yield-to-maturity | 12.57% | 12.43% | 12.41% | 12.31% | 12.46% | 12.52% | |
| 4 | Proportion | 0.52124 | 0 | 0 | 0 | 0.478761 | 1 | 1 |

Actually, if we take the YTM of the bonds out of the picture, it is noticeable that only six possible bond combinations exist for two-bond portfolios and not all 2 out of 5

The answers in B4 through F4 and also in G3 are all the same as Khoury has calculated. We can see how Solver only chose two out of five bonds in order to maximize the average yield. If we only take a look at the different yields, we can easily find out that the bonds with the two highest yields were chosen to be in the portfolio. Yet, we have to note that one has duration less than and the other greater than the time horizon which is 3.2913 years. The answer that Solver has calculated raises two questions. (1) Is it a matter of coincidence that only two bonds were chosen for the portfolio? (2) Does duration have an impact on the answer or is the answer only based on the yield-to maturity? By the time we answer those two questions, we would reach our objective in identifying the portfolio that gives out the best results, that is if the intention is immunization.

Let us first try to answer the second question. In order to do that, let us raise the yield of bond D to 12.48%. This new yield for bond D is actually greater than the yield of

bond E. Yet, the answer does not change and the optimal solution is the same. Actually, this reveals two facts. First, when Solver sets a proportion to a bond, it does not only take into consideration the yield-to-maturity of the bond, which is obvious, but it also take into consideration the duration of the bond. Second, the higher the duration of the bond, in proportion to the YTM, the better it pays off. Although bond D now has a greater yield than bond E, the duration factor has overcome the small difference in the yield. Consequently, if we now set the yield of bond D to be 12.5%, we realize that Solver will assign 62.2% proportion to bond D and 37.8% to bond A. The switching from bond E to bond D has lowered the proportion invested in bond A by 14.3% and has added that and the original proportion of 47.9% of bond E into bond D to add up to 62.2%.

Why did not this happen when we set the yield of bond D to be 12.48%?

Actually, if we take the YTM of the bonds out of the picture, it is noticeable that only six possible bond combinations exist for two-bond portfolios and not all 2 out of 5 possible combinations, which by the way is 10.⁹ The reason is because we need to have the time horizon in between the two durations of the bonds. These combinations are formed by pairing the following bonds: A&C, A&D, A&E, B&C, B&D, and B&E. In order to fulfill the duration criteria, every combination has one and only solution or set of proportions for each bond as the equation is of first order and no negative proportions are allowed. These fixed proportions of each combination are presented in table 3.2 on the next page.

⁹ All possible combinations of 2 out of 5 = $5! / 2!(5-2)! = 120 / 12 = 10$

Table 3.2
The Possible Two-bond Combinations of the Immunized Portfolio with Time
Horizon of 2.913

| Combination | Bond A | Bond B | Bond C | Bond D | Bond E |
|-------------|--------|--------|--------|--------|--------|
| A&C | 32.13% | | 67.87% | | |
| A&D | 37.80% | | | 62.20% | |
| A&E | 52.06% | | | | 47.94% |
| B&C | | 58.82% | 41.18% | | |
| B&D | | 64.69% | | 35.31% | |
| B&E | | 76.62% | | | 23.38% |

When we raised the yield of bond D to 12.48%, the combination of bond A and bond D would have given a weighted average YTM of:

$$0.378 \times 12.57\% + 0.622 \times 12.48\% = 12.514\%$$

Yet, the answer did not change as bonds A and E still formed the optimal solution with an average YTM of 12.52% as cell G3 shows in the picture.

However, a 12.5% YTM for bond D will give a weighted average YTM of 12.526% for the combination A&D hence making it the optimal solution. Actually, Solver also provides the sensitivity analysis of the proportions and this is displayed in table 3.3. Before Solver displays the results of the linear programming, a small window will appear whereby it will be enough to highlight the word “Sensitivity” in order to obtain the sensitivity analysis table. This window is presented in figure 3.6.

Table 3.3 entails that bond B will enter the solution if we increase its YTM by 0.18%, keeping other things constant. Similarly, bond C or bond D will enter the solution if we increase their YTM by 0.08% or 0.18%, respectively, keeping other things constant.

optimal solution for the problem thus more than two bonds can enter the solution. Yet, in every optimal solution, the number of bonds will be only two. In case we add a third major constraint to the linear programming problem, it will be possible to have three bonds at the same time in the optimal solution.

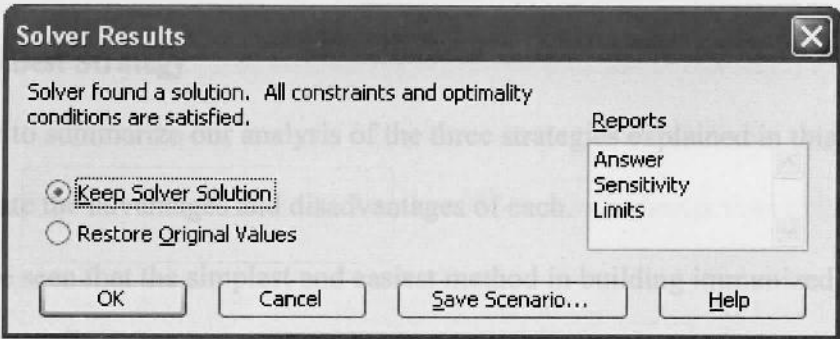
Table 3.3

The Sensitivity Analysis of the Bond Proportions as Calculated by Solver

| Cell | Name | Final Value | Reduced Gradient |
|--------|-------------------|-------------|------------------|
| \$B\$4 | Proportion Bond A | 0.521236913 | 0 |
| \$C\$4 | Proportion Bond B | 0 | -0.10% |
| \$D\$4 | Proportion Bond C | 0 | -0.08% |
| \$E\$4 | Proportion Bond D | 0 | -0.18% |
| \$F\$4 | Proportion Bond E | 0.478763087 | 0 |

Figure 3.6

Linear Programming Sensitivity Analysis Using Solver



In short, the combination with the highest weighted average yield to maturity will form the optimal solution of the maximization problem, keeping duration constant.

At this point, it is possible to conclude that the linear programming run by Khoury did not accidentally choose only two bonds as it will always be the case. However, it is possible to have alternative optimal solutions. This means that, there may be more than one

optimal solution for the problem thus more than two bonds can enter the solution. Yet, in every optimal solution, the number of bonds will be only two. In case we add a third major constraint to the linear programming problem, it will be possible to have three bonds at the same time in the optimal solution.

In this section, although our attempt was to make portfolios of three or more bonds, using linear programming did not serve that purpose as first place the program did not find optimality in the choosing of 3 or even more bonds.

Nevertheless, if a manager is still willing to consider three or more bonds for building an immunized portfolio, he/she will surely be using the option that we mentioned earlier in this section and that is to randomly set proportions till ending up by only two unknowns and calculating them. Although we have mentioned that such a decision is unprofessional, our analysis also proved that it does not contribute in the maximizing of the YTM of the portfolio.

D. Choosing the Best Strategy

In order to summarize our analysis of the three strategies explained in this chapter let us briefly restate the advantages and disadvantages of each.

We have seen that the simplest and easiest method in building immunized portfolios can be done by using zero-coupon bonds as their maturities and durations are the same. However, using figure 3.2, we have come to conclusion that for any given maturity, zero-coupon bonds pay less YTM than do coupon-bearing bonds. On the other hand, we also realized that choosing coupon-bearing bonds for the single-bond strategy can be time consuming, costly, and sometimes misleading. Alternatively, the selection process of the bonds in the two-bond strategy is relatively easy as it excludes complexities and deep calculations and totally depends on managers' perceptions and abilities in spotting

attractive bonds. We also have to mention that the two-bond strategy diversifies the portfolios' risks.

Finally, further diversification is possible by including a third or more bonds into the portfolio. However, by using Solver we were able to come to conclusion that this strategy is not applicable as we aim at maximizing the YTM of the portfolio. Applying such a strategy works out only if all the proportions are assigned randomly except for the last two which can be easily calculated.

From the above summary we can surely conclude that the two-bond strategy gives the best results and its usage is recommended and rewarding.

E. Fortifying the Best Strategy

Now that we have identified the two-bond strategy to be the best fit for immunized portfolios, let us take a step forward and study factors or choices that may further fortify these portfolios and provide better outcomes.

In the second section of this chapter we explained how the bonds' searching process is divided to two. Actually, we split the search into two intervals based on the bonds' durations. The lower interval was for bonds with durations shorter than the time horizon, and the upper interval was for bonds with durations longer than the same time horizon. Although the limit of the lower interval is known to be from 0 to less than the time horizon, there is no obvious limit for the upper interval as it seems to be open-ended. Actually, the limits of the two intervals need to be narrowed down especially that of the upper limit.

It is true that the lower interval is well defined; however, it is better for the durations in this interval to be closer to the time horizon for the following reasons.

First of all, we know from the yield curve that the higher the maturity of a bond, the higher the corresponding YTM hence the better the bond pays off. So, the yield curve makes it easier to realize that investing in higher maturity bonds is advantageous for investors seeking immunization.

Second, because bonds of the lower interval will mature prior to the date of the obligation, it is better to postpone their maturities for as long as possible. The reason is that the proceeds will be reinvested at yearly basis and in case interest rates decrease the end result will be harshly affected. For example, consider an immunized portfolio, call it A, for five years time horizon and including two bonds with maturities 2 and 7. The first bond will mature at year two and the proceeds will be invested yearly for three years. Now consider for the same time horizon that an immunized portfolio, call it B, holds two bonds with maturities 4 and 7. The first bond will mature at year 4 and the proceeds will be invested for 1 year. As proceeds are reinvested yearly for three years, portfolio A holds a greater reinvestment rate risk than portfolio B whereby the proceeds need only to be invested once.

If we hold on to the same argument and apply it on the bonds with durations longer than the time horizon, we would conclude that in order to maximize the portfolios' YTM it is better to take into consideration high maturity bonds. However, as we aim at bonds with higher maturities, we are actually increasing the duration of the bond to be selected for the immunized portfolio. Yet, the higher the duration of a bond, the less is its proportion in the immunized portfolio and the higher the interest rate risk, although the higher the YTM of the bond. To prove this, let us take a numerical example.

Two bonds with durations three and six would have respective proportions of $1/3$ and $2/3$ if the time horizon is five. For the same time horizon, if we substitute the six with ten, the proportion of the three-year duration bond will increase to $5/7$ and the ten-year duration bond will have a proportion of $2/7$ in the immunized portfolio.

horizon In conclusion, when searching for bonds with duration greater than the time horizon, the higher the maturity of these bonds, the less their proportions will be in the overall portfolio. Furthermore, we know that there is no linear relation between maturity and YTM. However, it is noticeable that the maturity of a bond increases faster than does its corresponding YTM whereby an additional year will only change the YTM by a smaller fraction. Therefore, as we increase the maturity of the bonds of the upper interval, the weighted average YTM of these bonds will decrease more than the increase in the weighted average of the bond of the lower interval.

outcome Again, to prove our point, let us use Solver in Microsoft Excel. For simplicity, let us only consider two bonds and apply few changes in their figures.

prove if We will use the bonds A and D from table 3.6. The combination of A and D results in a weighted average YTM of 12.52%. If we increase the duration of bond D to 4 and its yield by 0.2% to 12.7%, the weighted average would increase to 12.69%. However, if we attempt a second increase with the same amounts, the new duration of bond D will be 4.6 with a 12.9% YTM and an overall weighted average YTM of 12.64%. As we keep on doing the same for several more times, the portfolio's YTM will still gradually decrease. Actually, when we move from a high maturity bond to another with even higher maturity, the overall YTM will reach a peak point then start to fall as the increase in the maturity will not be compensated anymore by the slight increase in the YTM.

Although the diversification of immunized portfolio has been applied in different scopes and by different authors, our analysis in this chapter clearly states their various effects on the YTM of the portfolios. Finally, we have to mention that it is not necessary for managers to directly choose two bonds for building an immunized portfolio. Rather, managers may select several attractive bonds with durations close to their obligation's time

horizon and then to run linear programming using Excel or any other software that will choose two bonds out of all the selected ones (if the objective is to maximize).

The above conclusion will be certainly applied in the next chapter as we attempt to study the outcomes of different immunized portfolios. Moreover, our analyses in the next chapter will be supporting our current analyses as we will be directly examining the end results of immunized portfolios. Actually, we will use different combinations of two-bond portfolios which will have different weighted average YTM and with the use of the computer we will simulate every possible path of interest rates thus every possible outcome. Eventually, we will compare these different outcomes of each portfolio once with the desired outcome and then with the outcomes of the other portfolios. The results will prove if (1) immunization is valid and (2) if maximizing the YTM of the portfolio gives best results or not and/or if other criteria also need to be employed.

A. The Selection of the Bonds

Before starting our testing procedures, let us first list the bonds and their respective information that we will be using in our analyses. For our purpose, we will use real bonds from the U.S. bond market whereby we will get the information from the website investinginbonds.com. All the selected bonds are AA rating and above and they are

CHAPTER 4

TESTING IMMUNIZATION

How reliable is immunization?

The previous chapter presented preliminary safety measures that managers can make use of as they intend to do portfolio immunization. Taking those measures into account, we will be testing the validity of immunization as we will consider several immunized portfolios and expose them to fluctuations in interest rates. The end results of the portfolios will make clear if the theoretical claim that immunization shields investments against changes in interest rates is valid or not.

For our testing, we will use a special program in Microsoft Excel that simulates all possible paths of interest rates. Actually, we will simulate the paths for five years time horizon, and we will apply these paths on the identified portfolios to end up with all the possible outcomes. Also, we will analyze a variety of statistical summaries concerning the data stream of the outcomes of each identified portfolio. These analyses have two objectives. The first objective is to determine which portfolio is better secured from interest rate fluctuations and try to study the reasons for that, and the second objective is to evaluate the overall theory of immunization.

A. The Selection of the Bonds

Before starting our testing procedures, let us first list the bonds and their respective information that we will be using in our analyses. For our purpose, we will use real bonds from the U.S. bond market whereby we will get the information from the website investinginbonds.com. All the selected bonds are AA rating and above and they are

randomly selected. The purpose of selecting bonds with high ratings is to drop the default risk and its effect on immunization, which is one of the limitations that we mentioned in chapter 2. Table 4.1 represents the selected bonds from the cited website with their respective issuers, defined names, maturity dates, years to maturity, coupon rates, YTM, prices, and durations. We have to note that the years to maturity and the durations are calculated by using Microsoft Excel as this information is not found on the website.

Corporate Coupon-bearing Bonds Listed on www.investinginbonds.com as of May 5, 2005 with Adjustments in the Maturities and Durations

Table 4.1
Corporate Coupon-bearing Bonds Listed on www.investinginbonds.com as of May 5, 2005

| Issuer | Name | Maturity Date | Years to maturity | Coupon Rate | YTM | Price | Duration |
|------------------|------|---------------|-------------------|-------------|-------|-----------|----------|
| Wal Mart Stores | A | 8/10/2009 | 4.26 | 6.88% | 3.90% | 1114.522 | 3.70 |
| General Electric | B | 5/1/2008 | 2.99 | 3.50% | 3.54% | 998.79386 | 2.89 |
| Citigroup Inc | C | 3/6/2007 | 1.84 | 5.00% | 3.44% | 1027.1897 | 1.79 |
| Morgan Stanley | D | 4/1/2014 | 8.91 | 4.75% | 4.60% | 1010.6708 | 7.44 |
| Goldman Sachs | E | 7/15/2013 | 8.19 | 4.75% | 3.85% | 1062.0953 | 6.78 |
| Citigroup Inc | F | 8/27/2012 | 7.31 | 5.63% | 4.11% | 1094.0446 | 6.04 |
| General Electric | G | 11/21/2011 | 6.54 | 4.38% | 4.17% | 1011.5956 | 5.73 |

From table 4.1 we can realize that the terms-to-maturities are not round figures. Yet, for the simplification of the calculations we need to have discrete numbers. Therefore, let us form another table of the same bonds by only changing the maturities of the bonds and rounding them to the nearest number. Of course, the durations of the bonds will also be changing whereas the YTM and the coupon rate of each bond will remain the

same. It is also important to mention that as we slightly adjust the maturities of these bonds, their durations will automatically be adjusted yet more slightly. This can also be concluded by examining table 4.2 that displays the bonds with the mentioned adjustments in the maturities and the durations.

Table 4.2
Corporate Coupon-bearing Bonds Listed on www.investinginbonds.com as of May 5, 2005 with Adjustments in the Maturities and Durations

| Issuer | Name | Maturity Date | Years to maturity | Coupon Rate | YTM | Price | Duration |
|------------------|------|---------------|-------------------|-------------|-------|-----------|----------|
| Wal Mart Stores | A | 5/5/2009 | 4.00 | 6.88% | 3.90% | 1108.2054 | 3.65 |
| General Electric | B | 5/5/2008 | 3.00 | 3.50% | 3.54% | 998.79628 | 2.90 |
| Citigroup Inc | C | 5/5/2007 | 2.00 | 5.00% | 3.44% | 1029.6609 | 1.95 |
| Morgan Stanley | D | 5/5/2014 | 9.00 | 4.75% | 4.60% | 1010.8542 | 7.54 |
| Goldman Sachs | E | 5/5/2013 | 8.00 | 4.75% | 3.85% | 1060.9718 | 6.88 |
| Citigroup Inc | F | 5/5/2012 | 7.00 | 5.63% | 4.11% | 1090.8151 | 6.03 |
| General Electric | G | 5/5/2011 | 6.00 | 4.38% | 4.17% | 1010.9496 | 5.41 |

By the use of the seven bonds presented in table 4.2 it is possible to have 21 combinations ($7! / (2! \times 5!) = 21$) of two-bond portfolios. However, this number of combinations does not take into account the time horizon. Because the weighted average duration of the portfolio has to be matched with the time horizon, not every combination will be satisfying that criterion. If we set the time horizon to be 3, 4, or 5 we will only have 10, 12, and 12 combinations respectively. For the three years time horizon, all

combinations need to include either bond B or bond C as only these two bonds have durations less than the time horizon of three years. As for time horizons of 4 and 5, all combinations must include bonds A, B, or C exclusively. Tables 4.3 and 4.4 list all possible combinations for the three and five years time horizons respectively.

Table 4.3
All Possible Combinations for 3 years Time Horizon

| Combinations | A | B | C | D | E | F | G | YTM |
|--------------|--------|--------|--------|--------|--------|--------|--------|--------|
| A&B | 86.64% | 13.36% | — | — | — | — | — | 3.591% |
| D&B | — | 2.16% | — | 97.84% | — | — | — | 3.566% |
| E&B | — | 2.52% | — | — | 97.48% | — | — | 3.551% |
| F&B | — | 3.21% | — | — | — | 96.79% | — | 3.561% |
| G&B | — | 4.00% | — | — | — | — | 96.00% | 3.568% |
| A&C | 38.33% | — | 61.67% | — | — | — | — | 3.724% |
| B&C | — | — | 18.75% | 81.25% | — | — | — | 3.657% |
| E&C | — | — | 21.26% | — | 78.74% | — | — | 3.527% |
| F&C | — | — | 25.71% | — | — | 74.29% | — | 3.611% |
| G&C | — | — | 30.29% | — | — | — | 69.71% | 3.660% |

Table 4.4
All Possible Combinations for 5 years Time Horizon

| Combinations | A | B | C | D | E | F | G | YTM |
|--------------|--------|--------|--------|--------|--------|--------|--------|--------|
| D&A | 34.71% | — | — | 65.29% | — | — | — | 4.144% |
| E&A | 41.81% | — | — | — | 58.19% | — | — | 3.880% |
| F&A | 56.82% | — | — | — | — | 43.18% | — | 4.017% |
| G&A | 76.74% | — | — | — | — | — | 23.26% | 4.104% |
| D&B | — | 45.28% | — | 54.72% | — | — | — | 4.022% |
| E&B | — | 52.79% | — | — | 47.21% | — | — | 3.705% |
| F&B | — | 67.19% | — | — | — | 32.81% | — | 3.921% |
| G&B | — | 83.70% | — | — | — | — | 16.30% | 4.064% |
| D&C | — | — | 54.55% | 45.45% | — | — | — | 4.073% |
| E&C | — | — | 61.86% | — | 38.14% | — | — | 3.694% |
| F&C | — | — | 74.82% | — | — | 25.18% | — | 3.938% |
| G&C | — | — | 88.16% | — | — | — | 11.84% | 4.079% |

preceding period. Mathematically, the number of paths is expressed by 3^n , where n is the

The first column in each table represents all the possible two-bond combinations and lists the names of these two bonds. If we select each combination and move horizontally in the table, we will find out the proportions of the bonds in that specific portfolio. Finally, the last column displays the weighted average YTM of the portfolios.

We realize from the tables of combinations that for each time horizon there is one and single combination that maximizes the YTM of the portfolio. Actually, as we mentioned in the previous chapter, this solution can be obtained by using linear programming. However, in order to widen our study and to be able to compare different portfolios with different combinations, we will not only choose the portfolio that maximizes the overall YTM, as we will also take into consideration all other possible combinations that table 4.4 provides. Therefore, we will only do tests on the five years time horizon and forgo any other possible periods.

B. The Paths of Interest Rates

The main analysis in this chapter is based on the fact that the course of interest rates is unknown. Therefore, we will identify each and every possible path that interest rates may have during the holding period of the investment or till the obligation becomes due, and we will apply each path on the portfolios that we identified earlier. As we mentioned earlier in this dissertation, every year, interest rates may follow one of three possible directions. Interest rates may increase, remain constant, or decrease. This said, every year, the already existing path of interest rates will be split into 3 new lengthier paths. In other words, for every new period, the paths will be longer by one year and the total number of possible paths will be three times as much as the number of the paths of the

preceding period. Mathematically, the number of paths is expressed by 3^n , where n is the number of years of the time horizon. Table 4.5 shows how fast the number of paths increases every time we increase the number of years by 1.

Table 4.5
Number of Possible Paths According to the Number of Years

| Years | # of Paths | Years | # of Paths |
|-------|------------|-------|------------|
| 1 | 3 | 6 | 729 |
| 2 | 9 | 7 | 2187 |
| 3 | 27 | 8 | 6561 |
| 4 | 81 | 9 | 19683 |
| 5 | 243 | 10 | 59049 |

From the above table, it is realizable that, already at year seven we will have more than 2000 paths for interest rates. However, in our study we will only use the five years time horizon so we will be dealing with 243 paths at maximum. Because immunization shields investment from fluctuations in interest rates, applying the 243 paths on an immunized portfolio must show satisfactory results in all cases.

C. The Special Microsoft Excel Program

In order to preserve time and effort in listing all possible paths of interest rates and in calculating the end wealth position of each portfolio for each path, we will make use of a particular program developed in Microsoft Excel and define it as the Special Program to Run Immunization in Microsoft Excel (SPRIME). Actually, the program needs little input yet provides a wide array of output ranging from displaying every possible path of interest rates to providing various statistical summaries that serve in evaluating the results.

The utilization of the program is very simple and needs little guidance. In fact, the workbook is made up of only three worksheets that have the following names: input, results, and statistics. The first worksheet, input, is where the information about the future

obligation, the selected bonds, and the fluctuations in interest rates need to be inputted. First, for the future obligation, we need to input the current date, the maturity date, the future value of the obligation, and the current discount rate. This information will automatically give the amount needed for the current investment. However, SPRIME will allow its user to make a choice between selecting the discount rate to be the weighted average YTM of the portfolio, the weighted average RY of the portfolio, or any other desired rate. Second, the selected bonds need to be entered in the program. Actually SPRIME will allow the inclusion of five different bonds that are identified as attractive investments or included by a managerial decision. The needed information for each bond is the current date, the maturity date, the coupon rate, and the YTM of the bond. The years-to-maturity, coupon payment, the price, and the duration of each bond are then calculated by the program. Note that SPRIME does not restrict including five selected bonds as even a single bond can be enough to move on. As the desired bonds are inputted, it is time to find out what proportions of which bonds maximize the YTM of the portfolio. For this, we use Solver from the tools menu and this option will result in choosing the two bonds that maximize the YTM of the portfolio and separately display the two bonds as bond A and bond B. The third category of input is related to the fluctuations in the interest rates and their corresponding probabilities. SPRIME allows its user to assign possible changes in interest rates that may occur each year. In fact, we need to input possible interest rates increases and decreases for as many years as the time horizon is (for our testing we will assume that all these changes are 0.5%). No further input is required as at this point the program will display the solutions on the sheets named results and statistics.

In fact, once the input page is complete, the second worksheet will generate every possible path of interest rates according to the inputted data, the possible end results of bonds A and B that were selected in the first sheet by Solver, and the total possible

outcomes of the immunized portfolio. Of course, the end results of the bonds and the overall portfolio vary according to each path of interest rates.

Finally, SPRIME provides statistical summaries on the last worksheet that describes the whole data displayed on the second sheet. In fact, observing all the possible paths and the outcomes can be rewarding yet not enough to be able to study the results and draw conclusions. Therefore, the statistical summaries provided by the program are sufficient to evaluate the process of immunization.

D. The Applied Discount Rate

We mentioned earlier in this chapter that the validity test of immunization will be carried on the basis of five years time horizon. However, we still need to define our target amount after that date and the discount rate to use in order to calculate the present value of the amount. We will simply assign the target amount to be \$1,000,000 five years from the current date. However, in order to discount this amount to its present value we need to determine the discount rate. Actually, the discount rate can be assigned or calculated in various ways. For example, we can use the 5-year US treasury notes rate that is considered to be the risk free rate. Also, it is possible to use the 5-year zero-coupon bond rate that does not hold any reinvestment rate risk. However, because immunization is used in order to shield bond investments against fluctuations in interest rates and to allow these investments to have end wealth positions similar to the outcomes they would have if the rates were flat, we need to take into consideration the percentage return of the portfolios. In other words, if we use the risk free rate (or the zero-coupon rate) to discount the obligation, for sure this rate will be less than the weighted average return of the portfolios as the bonds in the portfolios are riskier than US treasury notes (or zero-coupon bonds). Also, using a lower discount rate will assign a higher amount for the current dollar investment. This means that,

as the portfolios' YTM are higher than the discount rate (or zero-coupon rate) and as we assume that rates are flat, by discounting at the risk free rate we are assigning an investment amount higher than the amount that each portfolio requires in order to secure the desired outcome. In short, in order to calculate the dollar amount needed to invest in, we have to discount the obligation using the weighted average return of the portfolio. At this point we may consider the weighted average return of the portfolio to be its weighted average YTM. However, the YTM figure holds two assumptions: (1) all the bonds in the portfolio are held to maturity, (2) the proceeds from the maturity payments of the bonds are not reinvested. Because the portfolios that we will use in testing immunization hold bonds either maturing after or before the 5 years time horizon and these will be either reinvested or sold prior to their maturities, the weighted average YTM is not an accurate figure of the actual yield that the portfolio will have at the end of five years. Therefore, the realized yield on the portfolio better describes its actual return as we intend to reinvest some bonds and to sell others at year 5. In order to calculate the realized yield of the portfolios we will use the earlier mentioned equation of the realized yield as formulated by Guilford and Babcock (1975), which is expressed in terms of the bond's duration (d), the holding period (H), the YTM (K_d), and the average reinvestment rate (RR) as follows:

$$RY = \left(\frac{d}{H} \right) K_d + \left(1 - \frac{d}{H} \right) RR$$

We have to note that the above equation is for a single bond. However, in order to calculate the weighted average realized yield of a portfolio, we need to calculate the realized yields of the bonds in the portfolio, to multiply each by its corresponding bond proportion, and then to sum up these weighted yields. The weighted average realized yield (RY) of the portfolio will be expressed in the following way:

$$RY_p = X_1 \left[\left(\frac{d_1}{H} \right) K_{d1} + \left(1 - \frac{d_1}{H} \right) RR \right] + X_2 \left[\left(\frac{d_2}{H} \right) K_{d2} + \left(1 - \frac{d_2}{H} \right) RR \right]$$

Where :

RY_p = weighted – average – realized – yield (portfolio)

X_1 = proportion(bond1)

K_{d1} = YTM(bond1)

d_1 = duration(bond1)

X_2 = proportion(bond2)

K_{d2} = YTM(bond2)

d_2 = duration(bond2)

RR = average – reinvestment – rate

In the above equation, the YTM of the bonds, their durations, their proportions, and their holding periods are known. However, we still need to define the average reinvestment rate. Actually, because the average reinvestment rate is the same for both bonds and the portfolios are immunized, we do not need to define the reinvestment rate because, even if we use different numbers, the outcome will be the same. Let us prove this mathematically.

$$X_2 = 1 - X_1$$

$$RY_p = \frac{X_1 d_1 K_{d1}}{H} + X_1 RR - \frac{X_1 d_1 RR}{H} + (1 - X_1) \left[\frac{d_2 K_{d2}}{H} + RR - \frac{d_2 RR}{H} \right]$$

$$RY_p = \frac{X_1 d_1 K_{d1}}{H} + X_1 RR - \frac{X_1 d_1 RR}{H} + \frac{d_2 K_{d2}}{H} + RR - \frac{d_2 RR}{H} - \frac{X_1 d_2 K_{d2}}{H} - X_1 RR + \frac{X_1 d_2 RR}{H}$$

$$\frac{dRY_p}{dRR} = 0 + X_1 - \frac{X_1 d_1}{H} + 0 + 1 - \frac{d_2}{H} + 0 - X_1 + \frac{X_1 d_2}{H} = 1 - \frac{X_1 d_1}{H} + \frac{X_1 d_2}{H} - \frac{d_2}{H}$$

$$\frac{dRY_p}{dRR} = 1 + \left(\frac{1}{H} \right) (X_1 d_2 - X_1 d_1 - d_2)$$

However, $X_1 d_1 + X_2 d_2 = H$, thus, $X_2 d_2 - H = -X_1 d_1$

also, $X_1 = 1 - X_2$

$$\therefore \frac{dRY_p}{dRR} = 1 + \left(\frac{1}{H} \right) ((1 - X_2) d_2 + X_2 d_2 - H - d_2) = 1 + \left(\frac{1}{H} \right) (d_2 - X_2 d_2 + X_2 d_2 - H - d_2)$$

$$\frac{dRY_p}{dRR} = 1 + \left(\frac{1}{H} \right) (-H) = 1 + \frac{-H}{H} = 1 - 1 = 0$$

The result of these calculations entails that, no matter what we set the reinvestment rate to be, the weighted average realized yield of the portfolio will not change whatsoever. Of course, this is true because the holding periods and the reinvestment rates are the same for both bonds and the portfolio is immunized. We have mentioned earlier that immunization neutralizes the reinvestment rate risk and this is why any change in the reinvestment rate will not have an effect on the RY of the immunized portfolio.

Now that it is possible to calculate the weighted average realized yields of the portfolios, it will also be possible to identify the dollar investment needed for each portfolio. This information is summarized in table 4.6 as it lists the different portfolios with their corresponding weighted average YTM, weighted average realized yield, the difference between the two yields, and the dollar investment needed for each portfolio that is the \$1,000,000 discounted 5 years using the weighted average realized yield of the portfolio. $(\$1,000,000 \div (1 + \text{weighted average RY})^5)$.

Table 4.6

The Weighted Average Realized Yield of the Selected Portfolios

| Combination | YTM | RY | RY - YTM | Dollar Investment |
|-------------|--------|--------|----------|-------------------|
| D&A | 4.144% | 4.267% | 0.123% | \$811,460.50 |
| E&A | 3.880% | 3.872% | -0.008% | \$827,014.09 |
| F&A | 4.017% | 4.042% | 0.025% | \$820,273.66 |
| G&A | 4.104% | 4.120% | 0.016% | \$817,189.08 |
| D&B | 4.022% | 4.265% | 0.243% | \$811,540.81 |
| E&B | 3.705% | 3.766% | 0.061% | \$831,231.92 |
| F&B | 3.921% | 4.000% | 0.079% | \$821,945.43 |
| G&B | 4.064% | 4.106% | 0.042% | \$817,735.55 |
| D&C | 4.073% | 4.394% | 0.321% | \$806,526.56 |
| E&C | 3.694% | 3.789% | 0.095% | \$830,317.40 |
| F&C | 3.938% | 4.041% | 0.103% | \$820,301.58 |
| G&C | 4.079% | 4.132% | 0.053% | \$816,746.49 |

From the above table, we notice that the realized yields of the portfolios are different from their YTM especially for the combination D&C whereby the difference is 0.321%. Although the average change between the YTM and the RY is not large (average = 0.096%) with a range (largest value – smallest absolute value) of 0.313%, the last column in the table shows the significant effects of these differences whereby the range of the dollar investment is \$19,771.42. The latter entails that, although all combinations will have similar end wealth positions, the dollar investment varies among the different combinations. This being said, D&C is the combination that needs the least amount of investment thus at this stage this portfolio is the most favorable. However, it is too early to draw conclusions because we need to take into consideration all the possible outcomes of these portfolios, which is the objective of this study.

E. The Nature of the Data. *It is possible to change the probabilities of the possible*

movements In the next section of this chapter, we will use the computer program to calculate the possible outcomes of each of the 12 different portfolios that are listed in table 4.6. Yet, we are not interested in the second sheet of the program that displays the paths and the outcomes, as we will be examining the statistical summaries that the program provides on the statistics sheet. However, before going into the statistical part, we have to examine the nature of the data that will be generated by the program in order to be able to perform *the* statistical analysis. First of all, we have to mention that the data that is generated according to the interest rate paths is a part of a population. The population includes all outcomes *on* corresponding to every possible interest rate value with every possible path for a period of five years. This being said, the population includes an infinite number of outcomes. *comes* However, the generated data include the outcomes that correspond to the paths of interest rates whereby the change in interest rates between two consecutive years is either 0% or $\pm 0.5\%$. Also, these data have special attributes. For example each outcome corresponds to a unique interest rate path in a total of 243 paths. The uniqueness of the paths lies in the fact that, because we are considering 5 years time horizon and therefore we can only have 5 possible changes in interest rates, each path will have a distinctive direction for interest rates. For instance, if we have a path whereby interest rates move up, down, constant, down, and up respectively from year 1 to year 5, there will not be any other similar path with all the similar movements at the same years. Yet, we may have the exact opposites of paths in case these paths do not have constant rates between any two given years. That is, if we have a path where interest rates go up, down, up, down, and up we will surely have a path whereby rates go down, up, down, up, and down. If we draw these paths in a form of a tree, it will resemble a probability tree whereby the probabilities of the 243 outcomes are all the same and each is equal to $1/243$ and every movement in interest rates will have a

probability of 1/3. Of course, it is possible to change the probabilities of the possible movements in interest rates thus each outcome will have a different probability according to the interest rate path that it corresponds to.

Taking all the above facts into consideration, we can say that although the outcomes of the portfolios are not randomly selected, these data are representatives of the population and hence can be viewed as or considered to be a random sample.

When we examine the results of the portfolios, we will study the skewness and the kurtosis of the distributions and find out that the generated outcomes of each portfolio do not follow a normal distribution. However, according to the Central Limit Theorem, when we have a large sample, it is possible to assume that the means of these different large samples follow a normal distribution. The sample size that we are using is 243 outcomes for each portfolio thus it is possible to use descriptive statistics and hypothesis testing in order to study these outcomes.

F. The Results of the Portfolios and the Analysis

Before taking a look at the statistics of the outcomes, let us point out how these outcomes are calculated in Excel. For this purpose, let us set the current interest rate to be i_0 and the interest rate at year n to be i_n . Let us assume that the immunized portfolio includes bonds maturing in two years and other bonds maturing in six years whereby the time horizon is five years. The end value of the 2-year bonds at year 5 will be equal to:

$$C \times (1 + i_1) \times (1 + i_2) \times (1 + i_3) \times (1 + i_4)$$

$$(P + C) \times (1 + i_2) \times (1 + i_3) \times (1 + i_4)$$

where, C is the periodic coupon in dollars and P the principal value in dollars.

We have to note that the above two components have to be added and then multiplied by the total number of the corresponding bonds in order to obtain the final wealth. As for the six years maturity bonds, they will have and end wealth of:

$$C \times (1+i_1) \times (1+i_2) \times (1+i_3) \times (1+i_4) + C \times (1+i_2) \times (1+i_3) \times (1+i_4) + C \times (1+i_3) \times (1+i_4) \\ + C \times (1+i_4) + C + P_5$$

$$\text{where : } P_5 = \frac{P+C}{1+i_5}$$

where, P_5 is price of the bond at year 5.

The price of the bond will be added to the reinvested coupons and then multiplied by the total number of corresponding bonds in order to have the end wealth. As we add the end wealth of the 2-year and 6-year bonds, we will have the total outcome of the portfolio at year 5.

Table 4.7 summarizes all the statistical figures of the portfolios' outcomes as determined by SPRIME. These statistical summaries include the outcomes' (1) mean, (2) standard deviation, (3) minimum value, (4) maximum value, (5) range, (6) probability of having an outcome less than \$1,000,000, (7) probability of having an outcome above \$1,000,000, and (8) the Z value for hypothesis testing. Table 4.7 also includes all the information listed in table 4.6 in order to easily examine and compare the information of the various selected portfolios.

In chapter 3, we have mentioned that maximizing the YTM of the portfolio will have the most satisfactory end results and will better immunize the portfolio.

Table 4.7

Statistical Summaries of the Selected Portfolios

| Combination | YTM | RY | RY - | | Investment | Mean | Dev. | Minimum | Maximum | Range | Pr(less) | | Pr(above) | | Z |
|-------------|-------|-------|--------|------|--------------|-------------|---------|-----------|-------------|----------|----------|--------|-----------|--|---|
| | | | \$ | Std. | | | | | | | \$1M | \$ 1M | | | |
| D&A | 4.14% | 4.27% | 0.12% | | \$811,460.50 | \$1,000,350 | \$4,489 | \$992,650 | \$1,010,163 | \$17,513 | 48.97% | 51.03% | 1.22 | | |
| E&A | 3.88% | 3.87% | -0.01% | | \$827,014.09 | \$1,000,161 | \$4,019 | \$993,306 | \$1,008,282 | \$14,976 | 49.79% | 50.21% | 0.62 | | |
| F&A | 4.02% | 4.04% | 0.03% | | \$820,273.66 | \$1,000,092 | \$3,669 | \$993,526 | \$1,007,596 | \$14,070 | 50.21% | 49.79% | 0.39 | | |
| G&A | 4.10% | 4.12% | 0.02% | | \$817,189.08 | \$1,000,062 | \$2,729 | \$994,958 | \$1,005,648 | \$10,690 | 49.38% | 50.62% | 0.35 | | |
| D&B | 4.02% | 4.27% | 0.24% | | \$811,540.81 | \$1,000,482 | \$6,330 | \$989,292 | \$1,014,594 | \$25,302 | 48.97% | 51.03% | 1.19 | | |
| E&B | 3.71% | 3.77% | 0.06% | | \$8312,31.92 | \$1,000,223 | \$5,547 | \$990,423 | \$1,011,858 | \$21,435 | 49.38% | 50.62% | 0.63 | | |
| F&B | 3.92% | 4.00% | 0.08% | | \$821,945.43 | \$1,000,167 | \$4,653 | \$991,667 | \$1,009,902 | \$18,235 | 49.38% | 50.62% | 0.56 | | |
| G&B | 4.06% | 4.11% | 0.04% | | \$817,735.55 | \$1,000,078 | \$3,133 | \$994,159 | \$1,006,577 | \$12,417 | 49.38% | 50.62% | 0.39 | | |
| D&C | 4.07% | 4.39% | 0.32% | | \$806,526.56 | \$1,000,454 | \$8,158 | \$984,649 | \$1,019,581 | \$34,932 | 48.15% | 51.85% | 0.87 | | |
| E&C | 3.69% | 3.79% | 0.10% | | \$830,317.40 | \$1,000,272 | \$7,032 | \$986,536 | \$1,016,108 | \$29,573 | 47.33% | 52.67% | 0.60 | | |
| F&C | 3.94% | 4.04% | 0.10% | | \$820,301.58 | \$1,000,146 | \$5,528 | \$989,279 | \$1,012,371 | \$23,093 | 48.97% | 51.03% | 0.41 | | |
| G&C | 4.08% | 4.13% | 0.05% | | \$816,746.49 | \$1,000,079 | \$3,469 | \$993,163 | \$1,007,579 | \$14,396 | 48.56% | 51.44% | 0.36 | | |

However, if we use the information shown in table 4.7, we realize that although the portfolio D&A has the highest YTM, this portfolio does not have the highest mean outcome especially that all the mean outcomes of the different portfolios are close to each other. Also, if we examine the last column in table 4.7, we realize that the Z values are all less than 1.645 that is the $Z_{\alpha/2}$ at 90% significance level. Actually, the hypothesis testing is to find out whether the means are different from the desired outcome of \$1,000,000 or not. In this case, the null hypothesis will be $\mu = 1,000,000$ and the alternative will be $\mu \neq 1,000,000$. Because all the Zs in table 4.7 are less than $Z_{\alpha/2}$, we can say that we do not have sufficient proof to reject the null hypothesis therefore all the means are equal to the desired outcome of \$1,000,000.

It is also possible to use analysis of variance (ANOVA) in order to find out if there is significant difference between the means. For this, we use the data analysis tools in Microsoft Excel to obtain the results displayed in table 4.8

Table 4.8
ANOVA of the Means of the Outcomes Using Excel

| ANOVA | | |
|------------|----------|----------|
| F | P-value | F crit |
| 0.19275813 | 0.998016 | 1.791938 |

From the above table we realize that the F value is less than the F critical value and also the P value is greater than the 5% α , which both mean that there is no significant difference in the means of the different portfolio outcomes.

Because of these insignificant differences between the means of the outcomes and the desired outcome from one side, and the insignificant differences among the means of

the outcomes from the other, and in order not to study the validity of immunization by examining the results of each portfolio, let us first identify the most rewarding portfolio and define the criteria for its superiority, and then study whether that portfolio is immunized or not and eventually evaluate the theory.

In order to choose the portfolio that is best immunized, we cannot only rely on the Mean column in table 4.7 especially that there is no significant difference between the smallest and largest value of the means whereby the range is only \$288. Also, the probabilities of getting numbers greater than and less than \$1,000,000 are not significantly different between the portfolios whereby they are almost 50% each in all cases. Actually, we have to shift our focus to the standard deviation of the outcomes of each portfolio. As a definition, the standard deviation is a statistic that describes how tightly/loosely all the various observations are clustered/dispersed around the mean in a set of data. Also, the standard deviation is a measure of the amount of variation that might be expected between the actual outcome and the forecasted value. In other words, as we are simulating the data, the mean is the middle point or rather a point estimate of these data however, the actual outcome at year five may vary either ways from the mean. This variation in the actual outcome is referred to as the standard deviation. The higher is the standard deviation of the portfolio, the higher will be the probability of having the actual outcome away from the mean. Yet, immunization entails having an actual outcome close to the desired outcome which means that the best immunized portfolio will have its outcomes tightly grouped around the mean. In short, the standard deviation is the risk of not getting the desired outcome. Therefore, the higher is the standard deviation, the greater is the risk.

If we compare the standard deviations of the portfolios, we realize that the combination G&A has the lowest risk. This means that, as all the portfolios have similar mean results, in order to minimize the risk of having undesirable outcomes we have to invest in portfolio G&A that has the lowest standard deviation. Actually, by moving one standard deviation both ways from the mean, we will cover around 65% of the whole data. This is possible if the data is normally distributed. Shortly, we will examine the distribution of the data of each portfolio. For example, if we take the combination G&A and move one standard deviation in both directions we will include the data having values between \$997,333 and \$1,002,791. This interval can also be interpreted by saying that, if we randomly select an outcome out of the 243 outcomes of portfolio G&A, we have a 65% chance that the selected outcome will have a value between 997,333 and \$1,002,791. As we mentioned, this percentage can be applied on data with a normal distribution. In order to study the distribution of the outcomes of the portfolios, we will use histograms and hence visualize the dispersion of the outcomes from their mean. The appendix displays all the histograms of the selected portfolios.

From the appendix, we can conclude that not all the portfolios' simulated outcomes follow a normal distribution. In fact, D&B, E&B, F&B, D&C, E&C, and F&C have asymmetric histograms whereby the level of risk and the level of uncertainty of the actual outcome of these portfolios are both high. Also, most of these mentioned portfolios' histograms have high frequencies on their extremes thus the probability of getting very low outcomes is proportionally high. Actually, the histogram of a normally distributed data is bell-shaped and symmetric from the mean. Therefore, portfolios G&A, G&B, and G&C are more likely normally distributed and these portfolios actually have the lowest standard

deviations among all the other portfolios. However, it is not enough to visually analyze the histograms and in order to define whether the outcomes of the portfolios follow a normal distribution or not, we have to do the skewness and the kurtosis tests of these data and the values have to be between -2 and 2 in order to have a normal distribution. Actually, the -2 and 2 is the approximation value of critical t at the 95% confidence level that in fact is equal to 1.96. The skewness and the kurtosis tests of the outcomes of each portfolio are summarized in table 4.9.

Table 4.9
Test of Skewness and Kurtosis of the Distribution of the Outcomes

| Combination | Kurtosis | Skewness |
|-------------|----------|----------|
| D&A | -3.49294 | 0.55607 |
| E&A | -3.70107 | 0.376268 |
| F&A | -3.43547 | 0.322194 |
| G&A | -3.15925 | 0.234188 |
| D&B | -2.76113 | 0.5646 |
| E&B | -2.88902 | 0.405309 |
| F&B | -2.82699 | 0.329798 |
| G&B | -2.75606 | 0.234952 |
| D&C | -2.1822 | 0.626305 |
| E&C | -2.27142 | 0.480509 |
| F&C | -2.31395 | 0.371252 |
| G&C | -2.36855 | 0.254855 |

From the above table, we can see that although the test for skewness are all between -2 and 2, all the kurtosis figures are less than -2, which means that there is symmetry in the data however it is not normally distributed. Actually, this will not create a barrier for our study as we have already explained that according to the CLT it is possible to use various statistical tests.

In conclusion, because all the portfolios have similar outcomes, we surely have to examine the risk factor of these outcomes that can be realized by observing their standard deviations. Because the combination G&A has the lowest standard deviation, it is viewed as the most appropriate and the least risky immunized investment. However, not every portfolio manager is risk averse and a considerable portion of managers takes risks or has different perceptions. For example, with the information given in table 4.7, a manager may decide not to reduce the risk of having unfavorable outcomes yet to reduce the amount of the current investment and thus to choose the portfolio D&C. However, another manager may consider that the risk of the portfolio D&C is too high and seeks to reduce the risk by reasonably increasing the investment. Therefore, this manager may choose portfolio D&A which has considerably less risk but a \$5,000 higher investment. From these examples, we can conclude that there is no certain explicit criterion that we need to take into consideration when choosing the most appropriate investment as managers' perceptions are different as well as their financial positions, objectives, predictions, ability of taking risks, and the extent of their risk aversion that all contribute to making different decisions.

It is possible to examine the correlations between the RY, the YTM, the RY-YTM, the standard deviation, and the mean outcome. For this, we will use the SPSS program and the correlation matrix is displayed in table 4.10.

Table 4.10

The Correlation Matrix

| | RY | YTM | RY-YTM | Std Dev | Mean |
|----------------|-----------|------------|---------------|----------------|-------------|
| RY | 1.000 | 0.872** | 0.199 | -0.376 | -0.002 |
| YTM | 0.872** | 1.000 | 0.654* | 0.115 | 0.440 |
| RY-YTM | 0.199 | 0.654* | 1.000 | 0.811** | 0.882** |
| Std Dev | -0.376 | 0.115 | 0.811** | 1.000 | 0.810** |
| Mean | -0.002 | 0.440 | 0.882** | 0.810** | 1.000 |

** . Correlation is significant at the 0.01 level (2-tailed)

* . Correlation is significant at the 0.05 level (2-tailed)

The above matrix shows that the strongest relation is between the RY-YTM and the standard deviation or the mean. This means that, the most important criteria in selecting portfolios for immunization is the gap between the RY and the YTM. The lower the gap is, the better immunization will be.

Although we do not find any relation between the mean outcome from one side and the weighted average YTM or RY from the other, it is possible to find a relation between the weighted average RY and the difference between the RY and YTM from one side and the standard deviation of the outcomes from the other. In order to elaborate this we will use linear regression. Actually, let us run linear regression by setting the standard deviations to be the dependent variable (Y). On the other hand, we set the weighted average RY and the difference between the RY and the YTM (RY – YTM) to be the independent variables (X1 & X2 respectively). By using the data analysis tool in Microsoft Excel, we can write the regression line as: $Y = 28,261.196 - 627,147.1 \times X_1 + 2,223,392.7 \times X_2$

Relying on the above equation we can say that the higher the weighted average RY of the portfolio is, the lower the standard deviation will be. Also, the higher the difference

between the weighted average RY and YTM is, the higher the standard deviation will be. In short, a higher weighted average RY is favorable for the immunized portfolio but we have to make sure that the gap between the weighted average YTM and RY is not large. Although in table 4.10 we did not find a significant relationship between the RY and the standard deviation, combining the RY with the RY-YTM in the regression analysis shows significance thus the RY has to be taken into consideration when comparing different immunized portfolios.

Now, in order to see whether the above regression line is valid, we have to take into consideration a new bond and form new portfolios. Furthermore, we need to calculate the weighted average YTM and RY and the standard deviation of the outcomes and compare the actual standard deviation with the regressed one. The new bond that we will use has three years to maturity, pays 4.25% coupon, and has a YTM of 3.568% (this bond is an actual bond listed on www.investinginbonds.com). The bond has duration of 2.88 years and sells at \$1,019.08. Let us assume this bond to be bond H and form the following portfolios: D&H, E&H, F&H, and G&H. Table 4.11 provides the actual standard deviation of these portfolio vis-à-vis the regressed standard deviation.

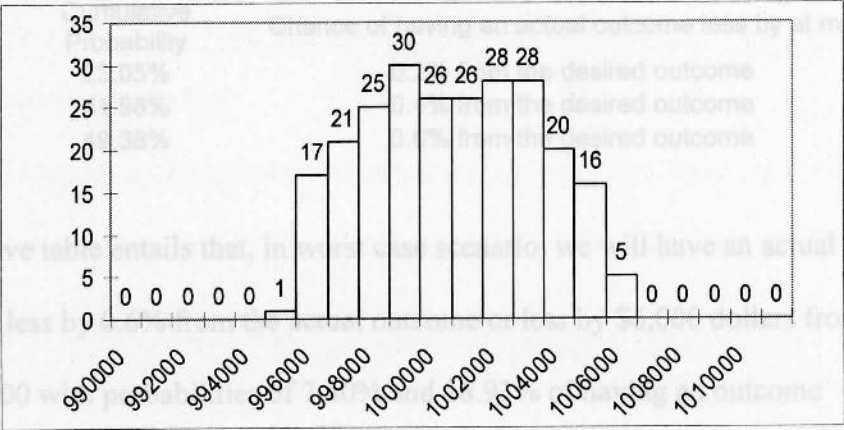
Table 4.11
The Actual Standard Deviation of the Portfolios Vis-à-vis the Regressed Standard Deviation.

| Combination | YTM | RY (X_1) | RY – YTM (X_2) | Actual (Y) | Regressed (Y) | Difference |
|-------------|--------|-----------------|-----------------------|---------------|------------------|------------|
| D&H | 3.695% | 3.761% | 0.066% | 6206.28 | 6141.6308 | 64.649244 |
| E&H | 3.718% | 3.774% | 0.056% | 5584.06 | 5837.7624 | -253.7024 |
| F&H | 3.758% | 3.797% | 0.039% | 4658.92 | 5315.5417 | -656.6217 |
| G&H | 4.068% | 4.109% | 0.041% | 3144 | 3403.3105 | -259.3105 |

From table 4.11 we can realize that there is a strong relationship between the independent variables and the dependent variable with insignificant error terms in the few hundreds. Actually, the adjusted R square, as displayed by SPSS, is 0.949, which means that 94.9% of the variations in the standard deviation are explained by the variations in the RY and the RY-YTM. So, we realize the weighted average RY plays a bigger role in the process of immunization than does the YTM solely.

However, we still need to answer the question that although the portfolio G&A has the lowest standard deviation, is the investment truly immunized? Let us determine few probabilities and percentages in order to evaluate immunization. Taking portfolio G&A into consideration, we will be able to calculate some descriptive probabilities by examining the extended version of the histogram of G&A that is displayed in Figure 4.1.

Figure 4.1
The Histogram of Portfolio G&A



than the In figure 4.1 we see that every bar has a value displayed above it which is the total number of outcomes corresponding to that interval. For instance, the values of 17 outcomes are between \$995,000 and \$996,000. From figure 4.1 we can conclude that we have a probability of 49.38% $((1+17+21+25+30+26) \div 243 = 0.4938)$ that the actual outcome of the portfolio will be less than \$1,000,000. However, we have a probability of 73.67% that the actual outcome will be greater than \$998,000. This means that, we have $100\% - 49.38\% = 50.62\%$ chance of having an actual outcome greater than \$1,000,000 plus $(26 + 30) \div 243 = 23.05\%$ chance of having an actual outcome between \$998,000 and \$1,000,000. The latter can also be translated into the fact that we have a probability of 73.67% that the actual outcome will be equal to, greater than, or less than by at most $(1,000,000 - 998,000) \div 1,000,000 = 0.2\%$ from the desired outcome. Table 4.12 displays all these probabilities.

Table 4.12

| Probability Table for the Outcomes of Portfolio G&A | | |
|---|------------------------|--|
| Probability | Cumulative Probability | Chance of having an actual outcome less by at most |
| 23.05% | 23.05% | 0.2% from the desired outcome |
| 18.93% | 41.98% | 0.4% from the desired outcome |
| 7.40% | 49.38% | 0.6% from the desired outcome |

The above table entails that, in worst case scenario, we will have an actual outcome at most less by 0.6% from the actual outcome or less by \$6,000 dollars from the desired \$1,000,000 with probabilities of 7.40% and 18.93% of having an outcome respectively less than 0.4% and 0.2% from the desired outcome. We realize that, although immunization does not give a 100% probability of having the actual outcome equal to the desired outcome, choosing the portfolio G&A entails that the actual outcome will be less

than the desired outcome by only 0.167% or \$1,670 on average. Because this average may be more than 2% for other portfolios, we have to realize that the selection of the bonds in constructing immunized portfolios has major effects on the actual outcome at the date of the obligation. We have enough evidence and measures to say that investing in the portfolio G&A will immunize the investment against fluctuations in interest rates for a period of five years. This does not mean that after five years we will exactly get the \$1,000,000 because the actual outcome has an average unfavorable error term of 0.167% which is highly insignificant.

It is recommended to apply immunization on portfolios taking into consideration composites of only two bonds, not less nor more, for the following reasons:

First of all, a single bond for the immunized portfolio creates problems in the selection process of the bond, in the maximization results of the portfolio's return, and in having greater risks. The single-bond strategy also adds problems to the rebalancing process in case the latter is applied.

Second, including multiple bonds in the immunized portfolio further diversifies the risk but also creates problems in the maximization process of the portfolio's return especially when linear programming is used.

It is preferable not to consider zero-coupon bonds in constructing immunized portfolios as these bonds pay less YTM vis-à-vis coupon-bearing bonds. Also, zero-coupon bonds will be automatically immunized if held to maturity.

CHAPTER 5

RECOMMENDATIONS

Using our analyses and their results throughout this dissertation, we will provide some recommendations that portfolio managers, investors, or institutions can take into consideration as they aim to immunize their investments.

It would be much rewarding to take into consideration the realized yield of the portfolios when setting the maximization criteria. However, it is also preferable that the gap between the weighted averages of the YTM and the RY to be small.

It is recommended to apply immunization on portfolios taking into consideration composites of only two bonds, not less nor more, for the following reasons:

First of all, a single bond for the immunized portfolio creates problems in the selection process of the bond, in the maximization results of the portfolio's return, and in having greater risks. The single-bond strategy also adds problems to the rebalancing process in case the latter is applied.

Second, including multiple bonds in the immunized portfolio further diversifies the risk but also creates problems in the maximization process of the portfolio's return especially when linear programming is used.

It is preferable not to consider zero-coupon bonds in constructing immunized portfolios as these bonds pay less YTM vis-à-vis coupon-bearing bonds. Also, zero-coupon bonds will be automatically immunized if held to maturity.

It is worthwhile to use a computer program, such as SPRIME, that simulates possible outcomes of different portfolios in order for the decision maker to have a clear view of the overall performance of these different portfolios, to study the risk associated with each portfolio, and to have enough statistical and probabilistic information in order to make the “best” perceived decision.

It is preferable not to have high maturity bonds in the immunized portfolio because these bonds will have greater durations, and obviously the higher these durations are, the riskier these bonds will be. However, given that the bond in the portfolio has duration shorter than the time horizon, it will be advisable to have these durations close to the time horizon.

In order to determine the amount of the current investment, it would be better to use the weighted average realized yield of the immunized portfolio in order to discount the future value of the obligation. Using the weighted average RY assigns a more accurate dollar investment value for immunized portfolios. Using a constant discount rate will significantly affect the outcome of these portfolios as each has a different weighted average return.

CONCLUSIONS

Although the actual outcomes of immunized portfolios highly depend on the course of interest rates, the selection of the bonds and their combinations in the immunized portfolios have major effects on the actual outcomes. Khoury (1983) mentioned that a more realistic approach to immunization is to choose a portfolio with the highest YTM. However, the analysis in chapter 4 also recommends taking into consideration the RY of the portfolio whereby there is strong negative relationship between the risk of the portfolio and the difference between the RY and the YTM.

Actually, our analysis provides evidence that immunization does not totally neutralize the risk of the interest rate because there will always be an error term, however insignificant, in the actual outcome of the portfolios at the maturity date of the obligation. However, using special programs that simulate the possible future outcomes of different portfolios can help minimize the error term as much as possible in order for the actual outcome not to vary much from the desired outcome.

In this study, we have specifically examined the impact of the fluctuation in interest rates on the outcomes of the immunized portfolio and came to conclusion that although there will always be an error between the actual and the desired results, the insignificance of the error term reassures the fact that immunization shields bond portfolios from the interest rate risk.

Noticeably, this study did not take into consideration the rebalancing of the immunized portfolio as has been repeatedly confirmed by a large body of literature to be

one of the problems of the theory, yet, our analysis gave proof that immunization is still workable. Therefore, if we consider rebalancing the portfolios that we tested in chapter 4, we would even come closer to our desired outcome and further verify that the doubtful claim of Stewart Coutts (1993) is biased whereby it focused on the assumptions and limitations of the theory without considering its abilities in removing the risk factor from bond investments.

In chapter 4 we compared the different means of the portfolios by using the ANOVA test. However, we did not find significant difference between the means. This means that immunization is robust and insensitive to changes in interest rates.

Finally, we can say that bond portfolio managers play a major role in having immunized outcomes especially that the analysis in this dissertation shows the significant impact of the bond selection process on the overall performance of the immunized portfolio.

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APPENDIX

The Histograms of the Portfolios Outcomes

