CRITICAL THINKING, MATHEMATICS CLASSROOMS, AND SCHOOL CULTURE: VIEWS OF PRINCIPALS, HEADS OF MATHEMATICS DEPARTMENTS, AND SECONDARY MATHEMATICS TEACHERS IN LEBANON

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By Taline Yessaye Hawatian

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Haigazian University

December 2019

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I would like to dedicate this thesis to the memory of my father whose strong belief in higher education always guided my path. I am also thankful for my mother's unconditional support and encouragement to accomplish my goal.

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ABSTRACT OF THE THESIS OF

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Major: Educational Administration and Supervision

Title: Critical Thinking, Mathematics Classrooms, and School Culture: Views of Principals, Heads of Mathematics Departments, and Secondary Mathematics Teachers in Lebanon

The purpose of this study was to explore the views of principals, heads of mathematics departments (HOD), and mathematics teachers of grades 9 to 12 in secondary schools in Lebanon about the role of critical thinking in mathematics classrooms. The study also investigated how principals and heads of mathematics departments viewed their role in creating a school culture conducive to critical thinking. Both constructs of critical thinking and school culture have been investigated at length by researchers, but the interplay between the two constructs has been rarely addressed. School culture was investigated based on Schein's three levels of cultural analysis: artifacts, espoused beliefs and values, and basic assumptions. The study adopted a qualitative research design, with multiple-case study approach. Questionnaires were administered to the participants and semi-structured interviews were conducted with them. In addition, several mathematics classrooms were observed. Data was coded, patterns were derived from the codes, and themes were emerged. The study identified three themes, namely: the Lebanese mathematics curriculum and official examinations do not foster critical thinking; an epistemological orientation based on student-centered exploration of knowledge and debating controversies create a school culture conducive to critical thinking; a shared vision of critical thinking as an educational goal, teachers' strong knowledge in critical thinking and its instructional strategies, professional development and training in teaching for critical thinking create a culture conducive to critical thinking. This study contributes to the literature by studying the relationship between two important educational constructs: critical thinking and school culture and showing how a school culture characterized by shared vision, collaborative leadership, and an epistemological orientation based on divergent thinking foster students' critical thinking. Moreover, this study makes recommendations to stakeholders and school leaders on how to create a school culture conducive to critical thinking by empowering teachers and providing them professional development in teaching for critical thinking.

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Chapter 1

Introduction

Research Problem

Promoting students' critical thinking through mathematics and creating a school culture conducive to critical thinking are naturally interwoven and related. Paul (2005) defines critical thinking as "the art of thinking about thinking in an intellectually disciplined manner" (p. 28), that passes through three phases: analyzing, assessing, and improving thinking (Paul, 2005). The National Council of Teachers of Mathematics (NCTM) states that mathematics education in general aims to develop and nurture critical thinking of students by encouraging them to think critically in the mathematics classroom (NCTM, 1989). The Lebanese mathematics curriculum developed by the Center for Educational Research and Development (CERD) introduces mathematics as a proper field to develop students' critical thinking, and it aims to achieve this goal through the construction of valid arguments, their evaluation, and the development of mathematical reasoning (CERD, 1999). However, the document "National Strategy of Education in Lebanon" prepared by the Lebanese Commission for Educational Sciences and presented to the Council of Ministers in 2008, states that education in Lebanon, the curriculum, teaching practices, and activities do not develop students' critical thinking skills and attitudes.

Hence, after 20 years of the implementation of the new curricula in Lebanon that emphasized critical thinking as an educational goal in secondary school mathematics, the schools have not been successful in achieving this goal (CERD, 1999). Since the views of teachers have a deciding role in their choice of instructional practices and exercises (Henningsen & Stein, 1997), and school leaders' views about mathematics education affect teacher practices (Nelson & Sassi, 2000), then, after many years of curriculum reform, it is worthwhile to study the views of school leaders and teachers in secondary schools in

Lebanon about critical thinking. According to Ernest (1989), the successful implementation of changes in the mathematics curriculum; in this case making critical thinking an educational goal, depends fundamentally on teachers' beliefs about the nature of mathematics, and their beliefs about teaching and learning mathematics (Ernest, 1989). Many researchers have investigated and found positive correlations between teachers' belief systems and their teaching practices and instructional decisions (Ampadu, 2014; Hongboontri & Keawkhong, 2014; Pajares, 1992; Zakaria & Maat, 2012). Thus, to ensure the successful implementation of change in the mathematics curriculum, research should address teachers' views about developing students' critical thinking.

Consequently, if one of the goals of mathematics education in Lebanon is developing students' critical thinking; and schools are expected to realize this goal, then school culture should be studied because of the important role it plays in implementing and sustaining any educational reform. To adopt effective practices educators should understand the interdependent nature of school culture, because all practices are reinforced and sustained by the different aspects of that culture (Gill & Boote, 2012). School culture is described as the set of norms, beliefs, and values that are beyond conscious awareness and shape the unique character of each school (Deal & Peterson, 1990). School leaders, however, play an important role in reshaping school culture by first reading the existing culture (its values, history, beliefs), then determining the positive components that should be reinforced, and finally, trying to reshape it according to the vision of a good school (Deal & Peterson, 1990). Therefore, to create a school culture conducive to critical thinking, the views of the school leaders should be studied as well. In the present study, the views of principals and heads of mathematics departments are studied as school leaders.

This study focused on school mathematics as a context for teaching critical thinking. Research has confirmed that there is a reciprocal effect between mathematics and critical

thinking; that is, if mathematics is taught using certain instructional strategies like problem solving, investigation method, and real life problems, then there is significant improvement of students' critical thinking (Aizikovitsh-Udi & Cheng, 2015; Firdaus, Kailani, Bakar, & Bakery, 2015; Su, Ricci, & Mnatsakanian, 2016; Sumarna, Wahyudin, & Herman, 2017). On the other hand, if students' critical thinking is developed, then there will be some gain in their mathematics achievements, especially in understanding complex mathematical concepts (Chukwuyenum, 2013; Jacob, 2012). These studies were conducted in secondary schools or institutions of higher education.

Purpose

The purpose of this research is to study the views of principals, heads of mathematics departments, and mathematics teachers in secondary schools in Lebanon on the role of critical thinking in mathematics classrooms, and school leaders' views of their role in creating a school culture conducive to critical thinking. Throughout the years, the term school culture has been defined and explained in many different ways: it is sometimes viewed holistically as an entity that encompasses and influences everything and everyone at school, or seen as the result of the interaction of different sub-cultures co-existing in harmony or conflict in a school setting (Prosser, 1999). Prosser (1999) defined school culture as "an unseen and unobservable force behind school activities, a unifying theme that provides meaning, direction, and mobilization for school members. It has both concrete representation in the form of artifacts and behavioral norms, and sustained implicitly jargon, metaphors and rites." (p.13).

However, other researchers like Heck and Marcoulides (1996) tried to study different observable aspects representing school culture like organizational structure, organizational values (time, innovation, participation), managerial processes (resources, responsive, leadership), organizational climate (teachers' perceptions), and teacher attitude. They

suggested that improving these aspects can have positive impact on school performance; furthermore, they argued that leadership is not the only variable that influences school culture, but it plays a significant role since it indirectly affects all the other variables that shape it (Heck & Marcoulides, 1996). As to teachers' beliefs about their school culture, according to Hongboontri and Keawkhong (2014), there is a reciprocal connection between teachers' beliefs, actions, and practices on one hand, and the school culture on the other; hence, to understand the school culture, teachers' beliefs, behaviors, and instructional practices should be investigated, and their common understanding of their school culture revealed. Thus, to better understand school culture, it is relevant to examine both principals' and teachers' beliefs about it.

Research Questions

1. How do principals, heads of mathematics departments, and mathematics teachers of grades 9 to 12 in Lebanon view the role of critical thinking in mathematics classroom?
2. How do principals and heads of mathematics departments view their role in creating a school culture conducive to critical thinking?

Rationale

Both constructs of critical thinking and school culture have been studied and investigated at length by researchers; but to the best of my knowledge, the relationship between the two educational constructs has been rarely addressed. Research that relates school culture to the development of students' critical thinking and whether certain types of school cultures are conducive to fostering critical thinking, especially in the context of secondary schools in Lebanon, is an understudied area of research.

Educational research has extensively addressed issues about critical thinking (Ennis, 1985; Lipman, 1988; Magno, 2010; Paul, 2005; Siegel, 1985), critical thinking and

mathematics (Aizikovitsh & Udi, 2015; Chukwuyenum, 2013; Firdaus et al., 2015; Jacob, 2012), perceptions of teachers about critical thinking (Alazzi, 2008; Choy & Cheah, 2009), dispositions of mathematics teachers about critical thinking (Innabi & El Sheikh, 2007), school culture (Deal & Peterson, 1990; Hallinger, 2003; Hargreaves, 1999; Prosser, 1999), and school culture, mathematics, and critical thinking (Gill & Boote, 2012). However, there is a gap in the relevant literature on the views of school leaders about their role in creating a school culture conducive to critical thinking in the Lebanese context. Tsui (2000) states that there is a compelling evidence linking college campus culture with the development of students' critical thinking, and further research is needed to get a deeper understanding of the phenomenon. Tsui's research was conducted in college campuses, and there is the need to investigate this relationship within the context of secondary education.

The following study is situated within the context of the mathematics department. According to Aizikovitsh-Udi and Cheng (2015), if mathematics teachers apply mathematics to real life problems that require higher order thinking, debate, and inquisition, then they will create classroom cultures that foster critical thinking, give students a chance to practice critical thinking, and encourage them to adopt these thinking habits beyond the mathematics classroom. Thus, mathematics classrooms have the potential to foster the critical thinking of students, and it is significant to study the views of principals and heads of mathematics departments on their role in creating a culture conducive to critical thinking in mathematics classrooms in secondary schools in Lebanon.

This study investigated the views of school leaders (principals and heads of mathematics departments) about critical thinking in mathematics. Their views about what counts as mathematical knowledge, how mathematics is learned, the nature of student engagement, and the teaching practices shape their judgments about the quality of teaching in the mathematics classroom (Nelson & Sassi, 2000), hence determining how their actions

will affect teachers' efforts in assisting students' construction of knowledge. Thus, it is essential that their views about teaching mathematics for critical thinking be studied, since these views will later influence teacher practices. However, it is not sufficient to study only school leaders' views about teaching critical thinking in mathematics, the views of mathematics teachers should be explored as well.

Henningsen and Zebian (2003) have studied higher order thinking, reasoning, and communication in the elementary mathematics classrooms in Lebanon by conducting direct observations of mathematics classrooms and teacher interviews; it was imperative that similar research be conducted on secondary mathematics classrooms in Lebanon as well. Ennis (1985, 2015) states that critical thinking represents a practical and significant part of higher order thinking and as such should play a pivotal role in schools. When investigating how mathematics is being taught in secondary schools in Lebanon, it is vital that we study the views of mathematics teachers on the subject, because of the important role they play in creating a culture conducive to critical thinking. For students to engage in higher order thinking in mathematics classrooms, the teacher plays an important role on two levels: first, the appropriate choice of worthwhile mathematical tasks and second, consistently supporting students' cognitive activities by asking for meaningful explanations and helping them make meaningful connections (Henningsen & Stein, 1997). Since the teachers' and principals' belief systems often shape their practices in the classroom, then to understand how mathematics is being taught in secondary schools in Lebanon, we must address the views of teachers and school leaders.

However, to be able to teach critical thinking one must have a profound understanding of its definition, goals, skills, and dispositions (Choy & Cheah, 2009), hence a study of principals', department heads', and teachers' understanding of critical thinking in general, and critical thinking in mathematics in particular was essential. After 15 years of adopting

critical thinking as an educational goal in Jordan, a study conducted on secondary school mathematics teachers on their understanding of critical thinking, yielded several results. Teachers did not have a comprehensive view of critical thinking; showed restricted understanding of the purposes, goals, processes, and dispositions of critical thinking; their conceptions of critical thinking were narrow in scope, fragmented in structure, and lacked clarity and consistency (Innabi & El-Sheikh, 2007). The principal argument of the researcher was that for students to be able to think critically, they must be instructed; but for teachers to be able to teach critical thinking, they should be cognizant in the conception of critical thinking.

Similar studies were also conducted on social studies teachers in Jordan (Alazzi, 2008) and teachers in Malaysia (Choy & Cheah, 2009), and yielded similar results. Thus, to understand how we can create a school culture that fosters critical thinking, we must have a clear view of the extent of teachers' understanding of the concept of critical thinking. This study sheds light on Lebanon to study school leaders' and teachers' views on teaching mathematics to develop critical thinking.

Gill and Boote (2012) argued that by trying to understand and explain mathematics education by focusing primarily and exclusively on teachers' views and individual perspectives about mathematics education, we miss the big picture; the interconnections that exist between the different aspects of a given culture, in this case school culture and mathematics classroom culture, and how they reinforce each other in creating a given culture.

School culture plays a central role in shaping the educational goals and practices of any given school, because school culture with its concrete presentations and implicit manifestations is the unseen force that governs school activities, gives purpose and direction to the school (Prosser, 1999). Since the beliefs and practices of teachers and principals on one hand play a role in shaping their school culture, and on the other are shaped by it (Heck

& Marcoulides, 1996; Hongboontry & Keawkhong, 2014), this research studied the views of principals, heads of department, and teachers in creating a school culture conducive to critical thinking, in the context of mathematics education.

Significance

The important contribution of this study is that it introduces a new perspective in studying the enhancement of students' critical thinking by shifting the focus to the culture of the educational institution. This research was conducted in the context of mathematics education in secondary schools in Lebanon, and hence added to the existing body of research about mathematics education in Lebanon. Similar research should be conducted in the context of other subject matter like sciences and social studies to gain a holistic view of teachers' and school leaders' views about fostering students' critical thinking and creating in schools the kind of environments that will encourage students to think critically.

Educators, policy makers, and stakeholders call for critical thinking to become a central educational objective in the Lebanese curriculum, but they rarely discuss whether the school culture in secondary schools in Lebanon fosters critical thinking. This study informs different stakeholders about the characteristics of a school culture that has the potential to enhance students' critical thinking according to the views of school leaders. It encourages them to contribute towards creating school cultures that motivate and empower students to think critically. Moreover, it informs stakeholders about the understandings of critical thinking that teachers and school leaders possess. Such understandings can help them plan carefully designed professional development activities, teacher preparation programs, and trainings.

Chapter 2

Literature Review

This study investigates the views of principals, heads of mathematics departments, and mathematics teachers about critical thinking in mathematics classrooms and the role of principals and heads of mathematics departments in creating a school culture conducive to critical thinking. The purpose of this chapter is to present a relevant literature review. The chapter is divided into the following parts: critical thinking, critical thinking and mathematics, school culture, school culture and mathematics, and school culture and critical thinking.

Critical Thinking

Throughout the years, scholars have defined critical thinking, summarized and captured its different components, skills and dispositions, and discussed its relevance in education (Ennis, 1964; Ennis, 1985; Ennis, 2013a; Ennis, 2015; Facione, 1990; Lipman, 1988; Magno, 2010; Paul, 2005; Siegel, 1985). Ennis (2015) defined critical thinking as "reflective and reasonable thinking that is focused on deciding what to believe or do" (p. 32), whereas Lipman (1988) defined it as self-correcting thinking which is based on criteria and is sensitive to context.

Scholars have often stressed the need for an operational definition of critical thinking that can be adopted in developing and enhancing students' critical thinking. In 1990, the Delphi consensus panel of forty-six experts in the field of critical thinking from the United States and Canada that was organized by the American Philosophical Association (APA) aspired to give a comprehensive definition of critical thinking. The panel defined critical thinking as "purposeful, self-regulatory judgment which results in interpretation, analysis, evaluation, and inference, as well as explanation of the evidential, conceptual, methodological, criteriological, or contextual considerations upon which that judgment is

based." (p. 3). Moreover, the definition elaborated the characteristics of a critical thinker focusing on both dispositions and skills of the critical thinker (APA, 1990).

Facione (1990) claims that this definition represents a consensus among the experts in the field of critical thinking. However, Johnson and Hamby (2015**)** in their meta-analysis of definitions of critical thinking state that one of the drawbacks of this definition is that it does not take a stand on the already established definitions of critical thinking that often include discrepancies, incompatibilities, and opposing views.

In recent years, the focus of defining critical thinking has shifted towards students' metacognition (Magno, 2010; Paul, 2005). Magno (2010) suggested that developing students' critical thinking could be facilitated by teaching them metacognitive strategies such as monitoring their own cognitive processes, progress, accuracy, and decision-making. Similarly, Paul (2005) defined critical thinking as "the art of thinking about thinking in an intellectually disciplined manner" (p. 28), that passes through three phases: analyzing, assessing, and improving thinking (Paul, 2005).

Critical thinking is one of the principal goals of education in secondary schools in Lebanon. Developing students' critical thinking entails guiding them to think rationally about their cognitive processes, to analyze and assess situations and information, to be able to make sound judgments and take decisions based on criteria and evidence. Critical thinking enables and empowers students to acquire substantive knowledge in any subject, retain that knowledge and make it a "permanent acquisition" of their mind (Paul, 2005). Principals, heads of departments, and teachers play a significant role in developing students' critical thinking, and researchers have studied their views and understanding of: critical thinking (Alazzi, 2008; Choy & Cheah, 2009; Howe, 2004), the role of critical thinking in mathematics (Innabi & El Sheikh, 2007), instructional strategies for teaching critical thinking (Abrami, Bernard, Borokhovski, Waddington, Wade, & Persson, 2015; Aizikovitsh-

Udi & Cheng, 2015; Firdaus et al., 2015; Jacob, 2012; Sumarna et al., 2017), and obstacles to teaching for critical thinking (Alazzi, 2008; Aliakbari & Sadeghdaghighi, 2013).

Critical Thinking and Mathematics

Critical thinking in mathematics. Scholars have studied the relationship between critical thinking and mathematics from different perspectives. Some scholars have studied whether developing students' critical thinking affects their performance in mathematics (Chukwuyenum, 2013; Jacob, 2012), whereas others have addressed the problem from the opposite direction by investigating how different approaches to teaching mathematics can enhance students' critical thinking (Aizikovitsh-Udi & Cheng, 2015; Firdaus et al., 2015).

Jacob (2012) found significant positive correlation between critical thinking skills and mathematics scores of first semester university students, hence concluding that developing students' critical thinking can help improve students' mathematical achievements, and going beyond the technical content of mathematics, teachers can create milieus that nurture critical thinking. Chukwuyenum (2013) examined the impact of teaching for critical thinking on performance in mathematics among senior secondary school students in Lagos state, and concluded that teaching critical thinking skills was an effective means to enhance students' understanding of mathematics concepts because the skills have helped in interpreting, analyzing, evaluating, and presenting data in a logical and sequential manner.

Su et al. (2016), introduced examples of how simple mathematical operations like multiplication can be used to enhance students' critical thinking, by discovering alternate methods of computation that simplify calculations and at the same time help the students understand mathematical concepts. According to Su et al. (2016), critical thinking skills can be practiced and enhanced through mathematics education, and at the same time developing students' critical thinking, like seeking alternative methods, identifying logical errors, in turn will enhance the students' problem-solving skills that are important for higher-level

mathematics. However, the results of a study conducted in Jordan showed that half of the mathematics secondary teachers could not explain how mathematics is related to critical thinking or give an example of a teaching strategy that develops critical thinking (Innabi & El-Sheikh, 2007).

Teachers' views of critical thinking. The main focus of studying mathematics teachers' views about critical thinking in mathematics is first, their attitudes to teaching critical thinking, whether they regarded it as their responsibility as mathematics teachers; second, whether they thought critical thinking can contribute to students' understanding of mathematics; and third, whether they are cognizant of how to teach mathematics for critical thinking. Even though most teachers thought that they could help their students become better critical thinkers, they did not think that critical thinking contributed to their students' mathematics knowledge (Innabi & El-Sheikh, 2007). Moreover, over a quarter of the teachers believed that they cannot help all students to become critical thinkers, in a country where critical thinking is an educational goal (Innabi & El-Sheikh, 2007).

Scholars have investigated teachers' definitions of critical thinking and its processes, as well as their views of the role of critical thinking in education. These studies have been conducted in different countries and within diverse educational systems. The only commonality between them was that all these institutions adopted critical thinking as an educational goal. Choy and Cheah (2009) investigated Malaysian teachers' definitions of critical thinking, students' ability to think critically, and teachers' role in incorporating critical thinking in their lessons. They concluded that even though teachers professed that critical thinking is an important element of learning and that they are teaching critical thinking to their students, in reality, they lacked a clear understanding of the concept of critical thinking. They also lacked the understanding about how to assess whether the students are thinking critically, often confusing it with one of the elements of critical thinking

like interpretation or argumentation, while totally ignoring other elements like reflection and appraisals (Choy & Cheah, 2009).

Similar research was conducted about teaching critical thinking in secondary education social studies in Jordan (Alazzi, 2008), and there are certain similarities in the results. The participants of the research, teachers of social studies in secondary classes, claimed that they teach critical thinking to their students in accordance with the teaching guidelines provided by their Ministry of Education. Those guidelines consider students' critical thinking as one of the goals of secondary education. The participating teachers could not give formal definitions of critical thinking, its skills and dispositions. Moreover, direct observations of their classes showed that they spent little or no time at all to develop students' critical thinking and were merely focused on covering the content (Alazzi, 2008).

A third study involves a comparative study on Canadian and Japanese teachers' views about critical thinking (Howe, 2004). The study concluded that although teachers of both cultures appreciated the important role of critical thinking, it was clear that they had not given much thought to its meaning and they had not discussed it with other teachers (Howe, 2004). According to Howe (2004), "critical thinking has become another educational buzzword" (p.522), in that policy makers and educators frequently use it but rarely discuss, study, or understand in depth. The thought-provoking aspect of this comparative study was that teachers' culture influenced how they related to critical thinking; for example, Canadian teachers concentrated on the cognitive domain of critical thinking, whereas Japanese teachers favored the affective domain (Howe, 2004).

A fourth study investigated secondary school mathematics teachers' definition of critical thinking after fourteen years of educational reform in Jordan and the results were similar to the previous two studies. In general, teachers' conceptions of critical thinking were narrow in scope, fragmented in structure, and lacking clarity and consistency. Furthermore,

the fourteen years of educational reform in Jordan, which aimed to develop students' critical thinking, did not produce any significant change in teachers' conceptions of critical thinking. Teachers' understanding was lacking in different aspects of critical thinking like its skills, dispositions, processes, requirements for teaching, and goals (Innabi & El-Sheikh, 2007).

It is essential to study educators' definition and understanding of critical thinking since they cannot teach for critical thinking unless they have a profound understanding of it. However, it is also vital to study how they view the role of critical thinking in education, and for the purpose of our study, specifically in mathematics education. Teachers' beliefs have a significant influence on how they acquire and interpret knowledge, how they choose appropriate tasks, teach course content, and monitor students' understanding (Pajares, 1992).

Instructional strategies that foster critical thinking. Critical thinking is one of the main goals of education in secondary schools and fostering and enhancing it is within the responsibilities of teachers of all subjects or disciplines and it is relevant to this study to investigate their views of the different instructional strategies that foster students' critical thinking. Abrami et al. (2015) conducted a meta-analysis of empirical evidence on developing critical thinking skills and dispositions in educational contexts. Empirical evidence supports the fact that students' critical thinking skills and dispositions can be developed in all classes and through all subjects by using effective instructional strategies (Abrami et al., 2015). According to this meta-analysis, two teaching strategies; dialogue (both whole class and small group) and authentic instruction, when combined with mentorship are the most helpful in fostering students' critical thinking (Abrami et al., 2015).

On the other hand, Marin and Halpern (2011) focused their investigation on the effects of embedded and explicit instruction in teaching high school students to think critically. The researchers reported that although students' critical thinking was enhanced in both cases, the gains were significantly greater for students who were taught using explicit

instruction. Marin and Halpern deduced that explicit instruction is an effective method for teaching critical thinking to high school students. One of the main concerns of teaching students to think critically is the transferability of critical thinking skills to students' everyday life in the future. The researchers argued that since explicit instruction used a wider range of diverse problems and focused on the structure of the problem rather than the content matter, then students will be able to transfer their critical thinking skills beyond high school to their everyday lives (Marin & Halpern, 2011).

Researchers have also investigated different instructional strategies of teaching mathematics for critical thinking. Aizikovitsh-Udi and Cheng (2015) argue that teaching strategies that aim to develop higher-order thinking skills improve students' critical thinking; that is, if mathematics is taught by applying it to real life problems, encouraging dialogue through investigative lessons, then students' critical thinking will be enhanced and a culture that fosters critical thinking will be established. Similarly, Sumarna et al. (2017), using a quasi-experimental design, studied the difference between learning mathematics using the mathematical investigation method and the expository method on the improvement of critical thinking skills of students, and concluded that mathematical investigation approach to learning improved students' critical thinking compared to the expository method.

Firdaus et al. (2015) studied the effects of using problem-based learning (PBL) module in mathematics on the students' critical thinking and confirmed that PBL enhances students' critical thinking skills in mathematics on three levels: identification and interpretation of information, evaluation of evidence and arguments. The researchers concluded that mathematics in addition to teaching mathematical content can also develop students' critical thinking skills, if it is taught using certain teaching strategies; for example, PBL. From these studies, it can be concluded that teaching mathematics using active learning

strategies based on problem solving, investigative approach, and dialogue enhance students' critical thinking.

Obstacles to teaching for critical thinking. Educators admit that critical thinking is an important educational goal but still fail in teaching for critical thinking, hence the necessity to study the barriers to teaching for critical thinking. Innabi and El-Sheikh (2007) based their study on the assumption that if students are to learn how to think critically, they should be taught by teachers that not only understand what critical thinking is but are also cognizant in how to teach for critical thinking. Hence, the first obstacle to teaching for critical thinking that is indicated in the literature is the educators' flawed or incomplete knowledge of critical thinking, its strategies, and how to incorporate it in their instruction (Alazzi, 2008; Aliakbari & Sadeghdaghighi, 2013; Innabi & El-Sheikh, 2007). The second obstacle stated in these studies is student characteristics and reluctance to think critically, and this attitude is justified by the absence of testing for critical thinking in official examinations (Alazzi, 2008). The third obstacle is curriculum requirements, delivery, and coverage of subject content (Alazzi, 2008; Choy & Cheah, 2009). The fourth obstacle mentioned in Jordanian studies refers to the role of schools in teaching students to socialize into their country's culture by accepting its norms and government and by avoiding the discussion of controversial issues (Alazzi, 2008). This attitude is in stark contrast with the strategies used in developing critical thinking. Evaluating controversies, questioning our beliefs, reflecting, and making judgment based on logic and evidence is an effective strategy for fostering critical thinking (Browne & Freeman, 2000).

The obstacles mentioned above are summarized from studies about teachers' views of critical thinking and teaching for critical thinking. In the relevant literature, to the best of my knowledge, school leaders' views about the obstacles to teaching for critical thinking have not been addressed, but some scholars have studied their attitudes towards higher order

thinking. School leaders have stated that one of the reasons for the failure of initiatives to foster students' higher order thinking is the apathy and resistance of school staff, in addition to the scarcity of resources and funds, and time restrictions (McCartney & Schrag, 1990). School leaders did not question the possibility that certain organizational and structural features of schools can also be responsible for the failure of these initiatives (McCartney & Schrag, 1990).

In the literature review, the failure to foster students' critical thinking is often explained by teachers' lack of knowledge of critical thinking, the strategies of teaching for critical thinking, and inadequate teacher preparation programs. Radulovic and Stancic (2017) critiqued this approach and the association of the development of critical thinking exclusively with instructional strategies, implementation of special programs, and teacher training and education. They believed that there are other obstacles to the development of critical thinking associated with the existing educational and social systems that require "questioning, deconstruction and reconstruction of status, role and power of pupils and teachers in the teaching process, but also in the process of curriculum development" (p.22). Hence, it can be concluded that the failure to develop students' critical thinking is not only associated with teacher preparation programs and appropriate curriculum changes but transcends to include organizational, structural, and social changes in schools. In order to be able to initiate these changes in an educational system we must understand the essence of a school, the school culture.

School Culture

After a review of the literature, it is apparent that school culture plays a pivotal role in school improvement (Hallinger & Heck, 2010), school effectiveness (Hargreaves, 1995), school performance (Hargreaves, 1995, Heck & Marcoulides, 1996), and school leadership (Hallinger & Heck, 2010). In fact, school culture can either hinder or help in the

implementation of any educational reform, improvement, or change. School culture is the educational construct that captures the unique character of a school by mirroring its values, beliefs, and traditions accumulated throughout the years; it is shaped by the interaction of all the key players in a school (Deal & Peterson, 1990). Scholars have defined school culture from different perspectives. Prosser (1999), presented two views of school culture. The first is a holistic view that regards school culture through the lens of effectiveness and success and believes that it affects everyone in schools and can be controlled to achieve educational objectives. The second view considers school culture as the product of the interactions among the different actors in a school that both affect and get affected by these interactions.

Regarding school culture, Schein (1992) introduced three levels of cultural analysis: Artifacts, espoused beliefs and values, and basic assumptions. Artifacts are the visible organizational structures and processes, the surface level, that are easy to observe and difficult to interpret. Espoused beliefs and values, the second level of organizational culture, refers to the strategies, goals, and philosophies of the organization. The deepest level of organizational culture is the basic assumptions, the unconscious, taken for granted beliefs, perceptions, thoughts, and feelings. According to Schein (1992) to get a deep understanding of a given culture, and to be able to predict future actions, the researcher must look beyond artifacts, and study the deeper levels of the culture by investigating the beliefs and values, and basic assumptions of the members of that culture.

By investigating the views of key players in secondary schools, the current study delves into the deeper level of their values, beliefs, and basic assumptions. Pajares (1992) claims that teachers' attitudes about any educational issue may be explained by the beliefs that the teachers hold; not only their epistemological, methodological, and efficacy beliefs, but also their personal beliefs about diverse issues like the nature of society, race, and family. These unexplored beliefs may explain why teachers continue to teach using old and

ineffective instructional practices despite many educational reforms (Pajares, 1992). Moreover, implementing change to create a culture that fosters students' critical thinking requires understanding of teachers' views and beliefs as the latter play a central role.

However, it is important to note that teachers' views are not the only factors that guide their behavior, even though they are important. This position is consistent with a study conducted by Gill and Boote (2012). The researchers report on the case of a teacher's (Ms. Bryan's) mathematics classroom, where she claims to believe in teaching mathematics for problem solving mirroring the values of curriculum reform, but in reality continues to teach for procedural proficiency in accordance with the school culture and her and the stakeholders' basic assumptions. This can be explained by the school culture and how, through a trickledown effect, it governs teachers' behavior and how they do things (Gruenert, 2000). Hence, this study investigates simultaneously teachers' and school leaders' views about critical thinking in mathematics and creating a school culture conducive to critical thinking.

Role of school leaders in shaping the school culture. The fact that school culture is often defined as "the way we do things here" has an underlying connotation that the school has an already established set of norms, values, and procedures; for any educational reform or change to succeed, it must be aligned with these norms and their underlying assumptions. However, the debate is whether school leaders shape the school culture or the school culture itself shapes and guides their performance.

Deal and Peterson (1990) argue that through utilizing different dimensions of the job as a symbol, potter, poet, actor, and healer, a principal plays an important role in shaping the school culture. This can be achieved by first reading the existing culture (its values, history, beliefs), then determining the positive components that should be reinforced, and finally, trying to reshape it according to the vision of a good school (Deal & Peterson, 1990).

Nevertheless, as the principal has an important role in shaping the school culture, so does the school culture in shaping the work and efforts of the principal, because initiatives that the principal takes to make changes in management processes, instructional strategies, and subject matter content are constricted by the school's current culture. Moreover, the findings of longitudinal studies confirmed that school culture usually has a greater effect on shaping the principal's work than the principal has on school culture (Hallinger & Heck, 2010). The researchers stressed the important role that leadership plays as a catalyst in influencing school improvement, but also reaffirmed that school culture must be considered as a key factor in bringing about any change or improvement (Hallinger & Heck, 2010). The principal as a school leader directly or indirectly plays a significant role in shaping the school culture and in creating a school culture conducive to critical thinking. Hence, their views should be studied.

However, it is important to note that in secondary schools, school culture is not only influenced by principals, but also by heads of departments (Abolghasemi, McCormick, and Conners, 1999; Smith & Szymanski, 1999). In fact, one of the characteristics of secondary schools that differentiate them from primary or middle schools is their highly departmentalized structure, where department heads often referred to as middle management have significant control over curriculum development, course content, teacher evaluation, and professional development (Siskin, 1991). Leithwood (2016) states that department heads as instructional leaders in secondary schools are in a powerful position to initiate change and improve teaching and learning, because of the characteristics that differentiate secondary schools, such as their large size, complex and specialized curriculum, and limited subject-content knowledge of the principals.

The role of the heads of departments in shaping school culture has been studied by many scholars especially in the context of secondary schools (Abolghasemi et al, 1999; Busher & Harris, 1999; Leithwood, 2016; Siskin, 1991; Smith & Szymanski, 2013). These

studies have revealed that in addition to their administrative responsibilities, heads of departments (HOD) play a central role as instructional leaders in shaping the school culture by aligning the teachers with the school mission, as well as by creating collegial and collaborative sub-cultures at the department level. Siskin (1991) states that to understand the sociocultural and organizational structure of high schools, researchers must study departments as fundamental units because in secondary schools' collegiality, collaboration, and school goals are best observed at this level (Siskin, 1991). Even though Siskin's research is based on American secondary schools, it is important to note that secondary education in Lebanon is also departmentalized and secondary teachers are similarly considered subject specialists.

School culture and mathematics. In an effort to explain the failure of the implementation of reform-based mathematics education in schools, Gill and Boote (2012) studied a teacher's (Ms. Bryan's) eighth grade mathematics classroom. They focused their analysis on five aspects of culture: language usage, standard practices, tools and equipment usage, ongoing concerns and values, and recurring problems. The researchers reported that none of these aspects by itself could explain the essence of Ms. Bryan's practice, but the interconnections among all these cultural aspects helped in understanding the teacher's behaviors. For example, in Ms. Bryan's case, her views of mathematics were generally procedural, and the school culture itself, manifested through the views of fellow teachers, administrators, parents, and stakeholders, also favored procedural mathematics. The researchers showed how the cultural elements could explain the teacher's persistence in teaching procedural mathematics despite her claims of teaching for problem solving. Hence, a teacher's views of mathematics and problem-solving are not the only factors that determine her practices, the interplay of school culture, mathematics classroom culture, and her epistemological views is also crucial. According to Gruenert (2000), to be aware of their

practice teachers must make "a conscious effort to get outside the existing culture and see things objectively without the lens of local norms or the shroud of tradition" (p.17). Therefore, in order to enhance students' critical thinking in secondary schools, in addition to studying teachers' views about critical thinking, it is also important to study the school culture.

School culture and critical thinking. In the relevant literature, there is minimum research on the role of school leaders in creating a school culture conducive for critical thinking. However, some researchers have investigated the role of school leadership in promoting higher order thinking, where critical thinking is regarded as an example of higher order thinking, in addition to, creative thinking and problem solving (McCartney & Schrag, 1990). According to McCartney and Schrag (1990), collegial school culture and instructional leadership are the key factors to fostering students' higher-order thinking. The researchers raise the importance of programs that promote higher-order thinking, strong leadership at the department level, principal's support to the HODs, teachers' ownership of the program, and creation of a culture of collegiality.

Tsui (2000) states that despite the fact that critical thinking is valued as an educational goal in higher education, the influence of college culture on development of students' critical thinking has been rarely explored. In the absence of similar studies in secondary schools, it is beneficial to review Tsui's study of the relationship between campus culture and critical thinking development. She based the study of the influence of college campus culture on students' critical thinking on three elements: epistemological orientation of the college (main method of knowledge acquisition), ability to instill responsibility and self-reflection, and the level of social and political awareness (Tsui, 2000).

According to Tsui (2000), the dominant epistemological orientation of a school determines to what extent students' critical thinking is enhanced. The preparation of

authentic critical thinkers requires "emphasizing analysis over recall, promoting active learning methods, encouraging collaborative exploration of knowledge, and setting students to work on questions and problems with no known or verifiable answers and solutions" (p. 436). However, her case studies showed that not only teachers received little encouragement for fostering critical thinking among their students, but they were sometimes punished for doing it. Moreover, students should have academic experiences to exercise their critical thinking beyond the scope of their studies, by participating in debates and discussions about social and political issues. Tsui (2000) concluded that "it is the confluence of unique combinations of subcultures that provides each institution with its distinct overall campus culture" (p.439).

In summary, the review of literature presented in this chapter showed: first, an overview of different definitions of critical thinking. Second, the literature investigates teachers' views about critical thinking, critical thinking in mathematics, instructional strategies of critical thinking, and obstacles to teaching for critical thinking. Third, the literature investigates the importance of school culture in initiating educational reform or change, and the role of the principal or HODs in shaping it. Fourth, the literature reviews the relationship between school culture, critical thinking, and mathematics.

Chapter 3

Methodology

This chapter presents the methodology of the present study which investigates the views of principals, HODs, and mathematics teachers of four secondary schools in Lebanon about the role of critical thinking in mathematics classrooms, and school leaders' views of their role in creating a school culture conducive to critical thinking. The study adopted an exploratory qualitative research design, using a multiple-case study approach. This chapter discusses the research design, the selection of schools and participants, the background of the schools, as well as data collection and analysis procedures used.

Research Questions

The study attempted to answer two research questions:

1. How do principals, heads of mathematics departments and mathematics teachers of grades 9 to 12 in Lebanon view the role of critical thinking in mathematics classrooms?
2. How do principals and heads of mathematics departments view their role in creating a school culture conducive to critical thinking?

Research Design

The current study is an exploratory study which used qualitative research methods to have an in-depth understanding of the problem. Little research has been conducted on how school culture can be conducive to the development of critical thinking, especially within the Lebanese high school context. Qualitative research is better suited for these kinds of studies that investigate problems not addressed previously and where all the variables are not known to the researcher beforehand (Creswell, 2003). After reviewing the relevant literature, Creswell (2003) summarized the characteristics of qualitative research as that being conducted in the natural setting, using multiple methods of data collection, being emergent

and not prefigured, as well as being interpretive in nature, holistic, and reflective about the biases and values of the researcher. This research was conducted in the natural setting of the study, the school; used multiple methods of data collection like questionnaires, semi-structured interviews, and class observations; and the themes that emerged from the study were not predetermined.

Based on Schein's (1992) definition of school culture that emphasizes the significance of basic assumptions and values in defining culture, it would be difficult to assess culture or investigate hidden assumptions using only questionnaires and quantitative methods (Maslowski, 2001). Moreover, since the study explores the views of principals, HODs, and teachers, then conducting qualitative research is justified because it is useful in exploring and understanding how individuals view and interpret social situations (Creswell, 2003).

In the current study the researcher aimed for an in-depth understanding of how principals, HODs, and teachers view critical thinking in mathematics classrooms and how the principals and HODs view their role in creating a school culture conducive to critical thinking. Multiple case study research design was used. A case study is considered to be an appropriate approach when the researcher investigates "how" or "why" questions, cannot control the variables, and focuses "on contemporary phenomenon within some real-life context" (p.1) (Yin, 2003). In a case study research design, the researcher explores in-depth an event or a phenomenon, that is bounded in time and situated within a context, using multiple methods of data collection (Baxter & Jack, 2008; Creswell, Hanson, Clark, & Morales, 2007). Yin (2003) listed five components of case study research design: the research questions, propositions (if any), units of analysis, linking data to the propositions, and criteria for interpretation of the results. However, Yin (2003) stated that in the case of an exploratory study, the research design may have a clear purpose rather than propositions (Yin, 2003). This study utilized multiple-case study where each school was the subject of an

individual case study. According to Miles, Huberman, and Saldaña, (2014), the precision and validity of the results of the research are strengthened when multiple-case study is used.

In this study, open-ended questionnaires and semi-structured interviews were used to study qualitatively and understand the meanings behind the school practices and staff behaviors. Yin (2003) considers open-ended interviews as a vital source of data collection in case studies, because "most case studies are human affairs" (p.92) that are best explored and explained through the lens of well-informed respondents that can add new and important perspectives to the studied issues. Moreover, to gather data and to gain additional insight on classroom interactions, the researcher observed mathematics classrooms (9 to 12) in the four schools. The classroom observations were held after the completion of the questionnaires and the interviews. The observations were guided by the research questions and looked for (in)consistencies between the teachers' stated views about teaching critical thinking in the mathematics classroom and real-life experiences in the classroom as well as teaching methods, the type of questions asked in the classroom, and the interactions between the teachers and the students or between the students.

This study is characterized by rich data collected from different sources, it is context-bounded and based on the interpretation of the researcher which contributes to the uniqueness of the study. The aim of the researcher is not generalizability, but in-depth understanding of the phenomenon.

Participants and Sampling

Four secondary private schools were selected as subjects for this multiple case study and purposeful sampling was used for school selection. The participants in the study were the principals, heads of mathematics departments, and mathematics teachers of the four schools.

Selection of schools. The four schools are non-free private schools with students from middle-high and middle socio-economic backgrounds. These schools have high rates of success in the Lebanese Baccalaureate examinations, which are held at the grade 12 level and administered by the Lebanese Ministry of Education and Higher Education. The schools fall under different categories of private schools: secular, sectarian, and foreign missionary, and may be considered representatives of their type of schools. The number of students in each school varied between 230 and 1500, while the class sizes varied between 14 and 33 students.

Background about the schools.

Case one. According to the school website, the school was established at the end of the nineteenth century by a charitable lady to help the Lebanese people suffering due to the civil strife in the country. It offers the Lebanese Baccalaureate as well as a Foreign Program that follows British based examinations for students that have exemption from Arabic studies. The school aims at the development of the child's whole personality with a special emphasis on moral and spiritual values. It is an inclusive school that welcomes students with learning difficulties and strives to help all students achieve their potential in a secure environment. Arabic and English are the languages of instruction and French is taught as a second foreign language. The school has more than 1500 students. The students belong to different religions, sects, nationalities, and races. The head of the mathematics department, in addition to administrative responsibilities, teaches three classes. The total number of mathematics teachers for grades 9 to 12 is five.

Case two. The school was established in 1930 in a rented building in Beirut with 15 students. The mission of the school was to provide the students with a solid education, cultivate a healthy national spirit, communicate the language and culture of the community, preserve the heritage, and prepare intelligent and qualified intellectuals to lead the community. By 1975 the school had more than 1000 students and had moved to a larger

building. Due to the civil war in Lebanon the school moved to different areas until it eventually settled in a newly built school complex in 1988. It offers the Lebanese and French Baccalaureate. Armenian, Arabic, and French are the language of instruction and English is taught as a second language. Today it has 500 students. The school body is homogeneous. The total number of mathematics teachers for grades 9 to 12 is two in addition to the head of the mathematics department.

Case three. The school was founded in 1985 in the Metn area and the first academic year of the school was in 1987. The mission of the school is to provide high quality education within peaceful Christian norms and spirit, at the same time teach the Armenian language, culture, and values to the new generation, and prepare responsible, devoted, and productive community members. The school offers the Lebanese Baccalaureate. The students learn Arabic, English, Armenian, as well as French as a fourth language. The school currently has 460 students. The school body is homogeneous. The total number of mathematics teachers for grades 9 to 12 is three in addition to the head of the mathematics department.

Case four. The school was established as a primary school in 1940, in a rural area in Lebanon and had 755 students. In 1970, the school obtained its license as a secondary school. The mission of the school is to educate younger generations to become conscientious, dedicated, morally strong citizens that uphold human and national values, in addition to academic excellence. The school offers the Lebanese Baccalaureate, the language of instruction is Armenian, Arabic, and English, and French as a fourth language. The school currently has 230 students. The student body is homogeneous. One teacher and the head of department teach the secondary classes.

Selection of participants. The researcher contacted by phone the principals of the four schools to ask for their preliminary approval to participate in the study. Later, the researcher met the principals in person to give the initial contact letter and explain the purpose of the study as well as the research procedures. The questionnaires were sent to the heads of the mathematics departments and the mathematics teachers through the principals. In three of the schools, the researcher met with the head of the departments and teachers to discuss the purpose of the study as well as arrange subsequent steps for collection of questionnaires, setting interviews, and observation dates. One principal insisted that the researcher meets only with the head of the mathematics department and coordinate with him the subsequent steps. Tables 1, 2, and 3 represent the demographic information of the participants.

Table 1

Demographics of Principal Participants

Principal	Gender	Administrative Experience	Education Level
S1P	Male	19 years	PhD in Physics
S2P	Female	2 years	MA in Science Education
S3P	Male	2 years	MA in Marketing
S4P	Male	14 years	BA in Armenian Studies

Note. S# denotes school number, P denotes principal

Table 2

Demographics of Head of Mathematics Department Participants

Head of Department	Gender	Teaching Experience	Administrative Experience	Education Level
S2H	Male	6	2	MA in Mathematics Education
S3H	Male	42	11	MA in Mathematics Education
S4H	Male	30	6	BS in Mathematics

Note. S# denotes school number, H denotes head of departmen

Table 3

Demographics of Teacher Participants

Teachers	Gender	Teaching Experience	Grade Level Teaching	Education Level
S1T1	Female	28	9,10, 11, 12	BS in Mathematics
S1T2	Male	16	8,9	BS in Mathematics
S1T3	Male	31	12	BS in Mathematics
S1T4	Female	11	8,9	BA in Business Administration
S2T1	Female	7	11	BS in Mathematics MA in Educational Administration
S3T1	Male	8	11, 12	MA in Mathematics Education
S3T2	Male	26	9	High School
S3T3	Male	4	10	MA in Mathematics Education
S4T1	Male	5	9	BS in Mathematics

Note. S# denotes school number, T# denotes teacher number

Data Sources

Three questionnaires were used in this study: the first, for secondary school mathematics teachers; second, for head of mathematics departments; and third, for school principals. Different questionnaires were prepared with different foci. The questionnaires for the mathematics teachers involved questions specifically about critical thinking and mathematics, as well as instructional strategies used in the mathematics classroom. Those addressed for principals and heads of mathematics departments targeted similar questions but from an administrative perspective. Moreover, their questionnaires also involved questions about creating a school culture conducive to critical thinking. The teachers were given the

choice of answering the questions anonymously or writing their names in case they would like to be interviewed later.

The first part of the questionnaires was related to the demographic information of the participants: gender, educational background, experience. The questionnaire prepared for teachers consisted of four parts: understanding of critical thinking, the role and importance of critical thinking in mathematics, the instructional strategies that can help improve students' critical thinking, and barriers to teaching for critical thinking. This questionnaire was prepared based on previous research studies like Innabi and El- Sheikh (2007) and Yacoubian (2016). The questions were adapted from these two research studies because of their relevance and immediate relation to the research question. The first study was conducted on mathematics teachers in Jordan, and hence its relevance to the current study since the context of the study is the mathematics department, whereas the second study was conducted in the Lebanese context and included, in addition to teachers' views about critical thinking, the views of school leaders as well.

The second and third questionnaires prepared for principals and HODs addressed their understanding of critical thinking and the role of critical thinking in mathematics in addition to their views on creating a school culture conducive to critical thinking. The questions related to school culture were based on the literature review and represented a synthesis of two approaches to studying school culture. The first approach is based on research conducted by Gruenert's (2005) six factors of school culture: unity of purpose, collaborative leadership, teacher collaboration, collegial support, professional development, and learning partnership. The researcher added three additional parts based on research conducted by Tsui (2000, 2002): epistemological orientation of the school, ability to instill responsibility and self-reflection, and the level of social and political awareness. A pilot study was conducted with

one teacher and one school leader to clarify vague terms and validate these two questionnaires.

After the completion of the questionnaires, the researcher conducted semi-structured interviews with the school leaders, that is, the principals and the heads of mathematics departments, to get an in-depth understanding of the participants' views about creating a school culture conducive to critical thinking. Interviewing individuals that hold higher positions in an organization is common practice in educational leadership research, because of the privileged information they have and their ability to influence the school (Brooks & Normore, 2015). The questions of the semi-structured interviews investigated and clarified the participants' definitions of critical thinking and views about developing students' critical thinking in mathematics. Simultaneously, the researcher investigated different aspects of school culture like unity of purpose (mission of the school), collegiality, collaboration, professional development, and tried to uncover the basic assumptions behind the values, norms, and school practices that govern the school. The researcher also conducted semi-structured interviews with secondary mathematics teachers so that they could either clarify or elaborate on their answers given in the questionnaires.

Two principals completed the questionnaires; the other two could not complete the questionnaires because they were busy but agreed to be interviewed. The researcher during these two interviews asked all the questions included in the questionnaires as well as questions to clarify or elaborate on some points. One head of department refused to participate, because she stated that she was only a senior teacher who facilitated running the mathematics department without making decisions related to school policy. The other three heads of mathematics departments completed the questionnaires and agreed to be interviewed. The nine participating teachers completed the questionnaires and eight teachers were interviewed. In total, four principals, three heads of mathematics departments, and nine

mathematics teachers participated in this research. The researcher collected 14 questionnaires (2 principals, 3 HODs, 9 teachers), administered 14 semi-structured interviews (3 principals, 3 HODs, and 8 teachers), and conducted 11 classroom observations (9 teachers and 2 HODs).

Data Analysis

The researcher analyzed the data for themes (Creswell, 2003). Data was analyzed by explanation building about the case in which the researcher tried to specify causal links and investigated plausible or rival explanations (Yin, 2003). At the first stage of data analysis, coding was used to summarize and classify similar data and identify repeated patterns (Miles et al., 2014). Sentences from interviews and questionnaires were divided into categories and labeled (Creswell, 2003). Later pattern coding was used to group these codes into smaller number of categories while trying to identify emergent themes and explanations (Miles et al., 2014). The researcher first analyzed the data within each case and later conducting cross-case analysis looked for similarities and differences between the cases to enhance generalizability. According to Miles et al. (2014), cross-case analysis deepens explanation and understanding, and enhances generalizability of the research findings. To enhance the trustworthiness of data analysis and to establish the construct validity and reliability of the study, the researcher relied on the use of multiple sources of evidence for data triangulation and the development of converging lines of inquiry (Yin, 2003).

Chapter 4

Research Findings

The purpose of this study was to explore the views of principals, heads of mathematics departments, and mathematics teachers of grades 9 to 12 from four private schools in Lebanon about critical thinking in the mathematics classroom and the role of principals and heads of mathematics departments in creating a school culture conducive to critical thinking. This chapter includes a summary of all the results that were obtained through multiple data collection methods in the sample schools. It presents data from the open-ended questionnaires, semi-structured interviews, and classroom observations.

Data are represented on a school by school basis. This chapter is divided into two parts: Views of the principals, heads of mathematics departments, and mathematics teachers about critical thinking and mathematics classrooms; and views of the principal and heads of mathematics departments about a school culture conducive to critical thinking. Each research question is stated, and relevant data grouped according to the conceptual framework. At the end of each part, there is a comparison among the schools leading into the presentation of the patterns. The chapter concludes with the presentation of the emerged themes.

The two research questions are:

1. How do principals, heads of mathematics departments, and mathematics teachers of grades 9 to 12 in Lebanon view the role of critical thinking in mathematics classroom?
2. How do principals and heads of mathematics departments view their role in creating a school culture conducive to critical thinking?

Critical Thinking in the Mathematics Classroom

Case one.

Principal's and teachers' definition of critical thinking.

Critical thinking involves analysis, reasoning, and logical thinking. Three of the four participating teachers, S1T2, S1T3, and S1T4, in their definition of critical thinking, stated that critical thinking included either analysis, reasoning, or logical thinking. One teacher, S1T4, during the interview, associated critical thinking with higher level thinking. None of the three teachers referred to metacognition; however, S1T1 indirectly mentioned it by stating that to think critically it was essential for students to think about "the way they are thinking" while solving problems. The principal, S1P, during the interview, stated that metacognition is considered the most important educational construct at the moment and, according to him, Lebanese teachers lacked knowledge in this aspect.

Critical thinking involves making decisions and judgments based on criteria. The principal believed that making judgments based on criteria is an essential component of critical thinking. Furthermore, he characterized critical thinking as the ability to identify and refute improbable answers based on certain criteria. The principal elaborated during the interview, "For example a student who thinks critically knows that the speed of a car cannot be equal to 3×10^{12}, because that number is greater than the speed of light".

Two of the four participating teachers included making decisions and judgments based on criteria in their definition of critical thinking. S1T1 defined critical thinking as "making reasoned decisions and judgments about what to do and think"; in other words, students consider the criteria or grounds for a thoughtful decision and do not simply guess or apply a rule without assessing its relevance. S1T4 wrote in the questionnaire that critical thinking means "to make logical judgments that are well thought of". On the other hand, S1T3 commented that critical thinking involved the evaluation of the calculated answers; that is, the ability to judge whether an answer is right or wrong.

Critical thinking involves problem solving. The principal associated critical thinking with solving problems that are challenging, related to real life, and basically different from what students are used to solve in class. During the interview, he defined critical thinking as "Critical thinking is one of two things, either the problem is different from what they have practiced or the problem is placed in the real world and they have to apply the things they have learnt in their lesson to an example in the real world. They would think how I can apply this when I am not even taught it. Critical thinking uses the same building blocks to build something new". Later the principal revisited this point and defined critical thinking as applying what you have learnt at school to solve problems you encounter when you go out to the world; hence stating the importance of transferability of critical thinking skills.

The four participating teachers agreed that problem solving is an essential part of critical thinking. Moreover, when discussing problem solving the four participating teachers associated critical thinking with solving problems that were not ordinary or standard, but problems that were "non-traditional", "difficult", "challenging", or "twisted"; that is, problems that differed from the standard exercises found in the textbooks that the students solved and practiced. Furthermore, the teachers stated that critical thinking is associated with the students' ability to use the knowledge they acquired at school to solve problems in the real world.

It is worthwhile to mention that three of the four teachers, S1T1, S1T2, and S1T4, in the questionnaires mentioned that the students' ability to identify and justify the steps required to solve a problem is an indication of critical thinking. One teacher, S1T4, stated that the students' ability to realize that a problem can be solved in more than one way is an indicator that the student is a critical thinker.

Dispositions of critical thinking. During the interview, the principal described a critical thinker as an open-minded person, who loves learning, has a great sense of

responsibility, persistent in his quest for answers, and not afraid to make mistakes. The principal thinks that a critical thinker does not admit failure in solving a problem but will break the problem into smaller parts and think about the next steps to get the answer. He identified a critical thinker as a "quite open-minded person who loves learning and has a quick brain", a person that "focuses on learning and not just obtaining good grades". He believed that a critical thinker has a great sense of responsibility; he said: "If you are responsible, then you are a critical thinker". A character trait that in the principal's opinion inhibits critical thinking is students' strong fear of making mistakes. He stated that "Students are terrified of making mistakes, they think about their marks".

The four participant teachers that completed the questionnaire defined critical thinking by choosing terms that refer to the cognitive skills of the student like analysis, interpretation, or inference, without any mention of the dispositions needed for critical thinking. However, when prompted further during the semi-structured interviews, they defined a critical thinker as a person who is motivated (S1T3, S1T4), eager to learn (S1T3), responsible (S1T2), persistent in looking for answers (S1T3), goal oriented (S1T1), loves to be challenged. As S1T3 suggested "critical thinkers are not necessarily smarter, but they love to analyze, ask questions, and persist till they find the answer". None of the teachers mentioned other dispositions of critical thinkers. S1T3 indicated that lazy and unmotivated students were not able to think critically.

All students can think critically. The four participating teachers agreed that all students can think critically, and the teacher plays an important role in enhancing students' critical thinking through "modeling the way of thinking", training, and practice. Nonetheless, after making this claim, subsequent inconsistencies were revealed in the teachers' responses during the interviews, in terms of different levels of critical thinking or students' innate ability to think critically.

S1T1 stated during the interview that "all students can think critically, but there are levels of critical thinking". S1T4 also expressed a similar view by stating in the questionnaire that students can be taught critical thinking "up to a certain level depending on the ability of the student". During the interview, S1T2 claimed that even though the teacher plays a role in enhancing students' critical thinking, he cannot teach the whole class to think critically, because "some of them will not get there". He also claimed that "some students are blessed by a good mind" and can think critically without any guidance. This sentiment was shared by S1T4 who said during the interview that "some students have an innate ability to think critically". S1T3 stated during the interview that "we cannot make all students think critically…some of them are not motivated to think, and just want everything to be easy".

One teacher, during the interview, stated that whether children are encouraged to think critically at home affects their attitudes towards critical thinking. Another teacher, S1T1, in the questionnaire stated that parents' socio-economic status has an effect on students' critical thinking. These two teachers were implying that school and teachers play an important role, but they are not the only factors that foster students' critical thinking. In general, the teachers believed that the development of students' critical thinking does not exclusively depend on teachers' efforts, but other factors like students' cognitive abilities and home environment also play a role in developing or hindering students' critical thinking.

Critical thinking is a continuous process. The principal's answers revealed that he thinks that critical thinking is a continuous process. During the semi-structured interview, he claimed that "I think critical thinking should be all the time in every lesson, in every single exercise…. You need to keep their minds active and agile all the time". He expressed the firm belief that all great teachers teach critical thinking whether they know the formal definition of critical thinking or not. At the same time, the principal was skeptical about the role of critical thinking in education nowadays. According to him, previously the focus of

educators was to teach critical thinking, but nowadays they are urged to provide students with a strong knowledge base, and "a strong knowledge base curriculum will prepare critical thinkers, people able to think critically". The principal's assumption is that if students are taught properly and have a strong knowledge base, then they will naturally become critical thinkers.

Two of the four participating teachers' responses revealed that they believed that critical thinking is not limited by subject content or time; it is a continuous process. S1T1 said "critical thinking is not an educational method; it is a way of life". This sentiment was also shared by S1T4 who suggested that critical thinking is a continuous process, not restricted by a class period, a lesson, or school. It may be concluded that two of the four participating teachers have a broad understanding of the conditions for critical thinking.

Teachers' formal education about critical thinking. The principal expressed his belief that the teachers have not engaged in professional development, attended workshops, or received training in teaching for critical thinking, and added "some of them probably do not know about critical thinking even if they teach it". The questionnaires revealed that the four participating teachers have not received any formal education in critical thinking during their university studies or professional development. S1T4 stated during the interview that "critical thinking is an expression that we often use, but never think about deeply or discuss". Two of the four participating teachers that completed the questionnaires, S1T1and S1T4, gave a formal definition of critical thinking using terms like reasoned decisions and judgments, criteria, logical judgment and justification, whereas the other two, S1T2 and S1T3, vaguely defined critical thinking as coming-up with ideas to solve problems and logical steps to reach a solution. However, one of the participants, S1T4, stated during the interview that the type of open-ended questions included in the questionnaire prompted her to read and conduct research on critical thinking before completing the questionnaire.

While discussing teachers' professional development, the principal, S1P, differentiated between two types of teachers: teachers that are eager to learn new methods to improve their teaching practices, and teachers that are reluctant to change and adopt new methods. The principal explained the teachers' reluctance to change by their refusal to admit that the methods that they have used for more than 20 years are not effective. The principal also added that teachers, in general, do not like taking risks and using new methods because innovations require extra time and effort.

In conclusion, participants' understanding of the definition of critical thinking is limited and based on their experience, rather than formal training or education. Moreover, their understanding included misconceptions as well as discrepancies like considering critical thinking an innate ability and its development limited by the students' cognitive abilities, or assuming that a strong knowledge base will naturally lead to critical thinking.

Teachers' views on critical thinking and learning mathematics.

Mathematics teachers play a role in enhancing students' critical thinking. In the questionnaires, the four participating teachers agreed that mathematics teachers play an important role in enhancing students' critical thinking while teaching mathematics. They stated that mathematics teachers can foster students' critical thinking by "modeling the way of thinking", training, and practice, as well as avoiding stating the answer of the problem directly, giving the student time to discover the answer, and guiding their progress by asking leading questions. Moreover, in the questionnaires, the four teachers agreed that teaching for critical thinking would help students learn mathematics in a better way.

Although both the questionnaire and semi-structured interview did not include any questions about the nature of mathematics, all participating teachers referred to it during the interviews. To explain the interrelationship between critical thinking and mathematics, the

teachers claimed that mathematics by its nature is abstract, not based on rote memorization, and requires higher level thinking. All the teachers claimed that thinking in general and critical thinking in particular, are essential to mathematics. S1T4 stated that "critical thinking can be applied to all aspects of life; however, we need it more in mathematics". During the interview, S1T1 stated that "you need critical thinking when solving math problems that involve more than one step". S1T2 in the interview said that "Math is based on thinking and if you are a critical thinker, then you can solve problems in all branches of mathematics". However, the same teacher later, during the interview, claimed that "some people can understand mathematics without being good critical thinkers".

Conceptual understanding of mathematics enhances students' critical thinking. The four participating teachers distinguished between two ways of doing mathematics: first, direct application of formulas and theorems; second, understanding the underlying arguments or justifications for a given answer and evaluating its relevance. They emphasized the importance of approaching mathematics problems in a "critical manner", going beyond the direct application of rules and formulas, by analyzing, assessing the relevance, and justifying each rule or step to reach the solution of the problem. Moreover, they associated this method of solving problems with critical thinking and considered applying formulas a non-example of critical thinking. S1T1 stated that "when they are just applying formulas, they do not need critical thinking". During the semi-structured interview, S1T1 said "when the teacher asks for the way of thinking, … this will make the student think critically and not just apply the formula". S1T4 in the questionnaire differentiated between types of students: "some students do mathematics by applying formulas they have mastered through practice, but they do not understand what they are doing. Other students can solve a problem and understand the use of a formula through logical thinking". In this instance, the teacher used logical thinking interchangeably for critical thinking, somehow equating the two types of thinking.

The classroom observations revealed that the main focus of the teachers was teaching mathematical procedures and practicing them in the form of extra exercises. The teachers constantly referred to the rules and theorems and asked for justifications but did not linger on each exercise to explain a concept; they rather tried to find the answer quickly.

Teachers' views on instructional strategies that foster critical thinking.

Classroom discussions and asking leading questions foster critical thinking. Three of the four participants' responses in the questionnaires revealed that the first strategy that they use to enhance students' critical thinking is asking leading questions to help the students clarify ideas, think, and analyze any given situation. All the participating teachers agreed on using the following two instructional strategies while explaining mathematical concepts or solving mathematics problems: first, asking leading questions to clarify ideas or to look for solutions to problems; second, asking students to give justifications. S1T1 said, "I use questions to lead them to the answer"; another teacher S1T2 stated in the questionnaire that the best instructional strategy to foster critical thinking is asking questions and giving students time to think and find the answer.

It can be concluded that the teachers have a constructivist approach to building knowledge. Nevertheless, during the semi-structured interview, when asked to describe a typical lesson, S1T3 said that he starts a new lesson by: first, introducing the new concept; second, giving the formal definition; third, stating the properties; fourth, giving examples and applications, all the time giving the students a chance to ask questions, solve exercises, and share answers. This method of instruction is not in lieu with the previously stated views.

The four teachers did not state that dialogue was an instructional method that they use to enhance the students' critical thinking, but they referred to asking questions that will encourage the students to think to reach a final answer. The classroom observations revealed

that the teachers ask short, convergent questions based on recall of properties and procedures previously used. They did not ask open-ended questions or thought evoking questions, only questions that required short and quick answers that paved the way to finding the final solution. Moreover, they called names and if the person could not answer, they moved to another student without probing further. The discussions that the teachers described were whole class discussions led by the teacher with no possibility of group discussions, because of the large number of students in each class (30 students), according to S1T2 and S1T4. None of the four participating teachers claimed to ask open-ended questions with a possibility of different answers; they were focused on getting the same answer using different methods of calculation or solution.

Asking challenging questions fosters critical thinking. The four participating teachers stated that they asked challenging questions to foster students' critical thinking; that is, problems that differed from the standard exercises found in the textbooks that the students solved and practiced. They characterized these questions as "non-traditional", "difficult", "challenging", "twisted", or "tricky". S1T2 said that "I give them tricks to solve using the rules and theorems they have studied". S1T4 stated that she asked "twisted" questions that trigger the thinking of the student and boost the overall critical thinking of the classroom. Moreover, she added that when solving problems, she models her way of thinking and encourages students to think in the same way. S1T3 said that to enhance students' critical thinking he asks, "questions apart from the book or expand the questions in the book".

However, in the questionnaire, when asked to describe a classroom activity they have carried out to enhance students' critical thinking, the four teachers failed to provide examples. Furthermore, two teachers S1T3 and S1T4 admitted that asking questions that enhance students' critical thinking was usually unplanned and spontaneous by adding parts to textbook questions or by "expanding questions" as one teacher put it. During the

observations conducted in the classrooms of the four teachers, none of them asked either challenging or difficult questions that required higher level thinking; most of the questions they asked were questions that facilitate the solution of the problem. Moreover, the exercises they solved were standard and similar to questions asked in official exams.

It is worthwhile to mention that the four participating teachers claimed that their textbooks did not include questions that foster critical thinking and the teacher has to prepare these questions themselves. Three of the four participating teachers stated that foreign textbooks, compared to Lebanese textbooks, included more questions that foster critical thinking. Two teachers, S1T3 and S1T4, commented that the American textbooks that they previously used included questions that enhance critical thinking, and S1T1 added that the A-level (British) textbooks also included such questions.

Solving authentic problems fosters critical thinking. The four teachers stressed the importance of solving real life problems and relating mathematical concepts to real life situations to promote students' critical thinking. According to the four teachers, mathematics should "be related to real life"' and teachers should "give examples from real life". However, when asked to provide examples that they have used where students used their knowledge in mathematics to solve real life problems, the teachers were hesitant to answer. S1T2 gave an example of calculating the height of a tree knowing the length of the shadow using Thales property, and S1T3 gave the example of using Pythagoras theorem to construct a room that is perfect square. Authentic instruction is defined by presenting students real problems that are meaningful to them, engaging, and that stimulate them to inquire. The examples that the teachers presented do not have the potential of interesting the students or engaging them in thinking. S1T2 stated that showing students pictures of him floating in the Dead Sea relates the concept of buoyancy in physics to real life experiences.

Teachers' views on barriers to critical thinking.

Students' lack of motivation and reluctance to think hinder development of critical thinking. All the teachers agreed that the attitude of the students towards critical thinking is instrumental to whether they aspire to develop their critical thinking or not. Although the teachers attributed students' failure to think critically to different character traits; students' lack of motivation and reluctance to think were a common trait. During the interview, S1T3 commented that "students want the final answer directly, quickly, without thinking hard". Similarly, S1T4 referred to their lack of perseverance by saying "they give up easily when faced with challenges". S1T1 explained some students' reluctance to think critically by an argument they make "why should I think if I can get what I want without thinking?"

Two of the four participating teachers, S1T1 and S1T4, pointed out that, the development of students' critical thinking is affected by their home life in terms of the extent of their exposure to critical thinking at home and the socio-economic status of the parents. On a similar note, one teacher, S1T1, argued that teacher's characteristics and their ability to think critically play a role in whether they can foster students' critical thinking or not. She wondered "If teachers are not using critical thinking in their lives, how can they teach it to their students?" Similarly, the principal reflected that teacher characteristics are important in fostering students' critical thinking; supportive teachers encourage critical thinking and teachers bullying students inhibits critical thinking.

Lebanese mathematics curriculum does not foster critical thinking. The principal commented that the Lebanese curriculum due to its "sheer volume" and "being tied by official exams" did not encourage the development of students' critical thinking, in addition to the negative impact of the official exams. Three of the four participating teachers S1T1, S1T3, and S1T4 stated that the Lebanese mathematics curriculum is essentially good, but it does not help develop students' critical thinking, whereas the fourth teacher S1T2 claimed that "critical thinking does not depend on the curriculum, but depends on the point the teacher

is targeting"; hence highlighting the importance of the teacher's role in fostering students' critical thinking. S1T3 argued that one of the drawbacks of the curriculum is that it has not been updated or revised for the last 20 years. Three of the four teachers agreed that the Lebanese textbooks do not include questions that foster critical thinking in contrast to American or British textbooks that include sections allocated to critical thinking problems. It is worthwhile to mention that while discussing the curriculum the teachers constantly referred to the textbooks.

The four teachers agreed that the Lebanese textbooks lacked problems related to real life. The teachers expressed their belief in relating concepts that students learn in the mathematics classroom to real life problems enhances their critical thinking, and this is missing in the Lebanese textbooks. S1T4, during the interview, described the Lebanese curriculum as "basically teaching the students the rules and when they are solving problems or exercises it is something abstract. It does not include enough problem solving or real-life examples". During the classroom observations, the researcher noted that teachers in general solved exercises included in the textbooks or extra exercises similar to exercises in the textbooks or official exam questions; during the observed lessons, the teachers solved neither authentic problems nor challenging questions.

The urgency to cover the content hinders the development of critical thinking. Two of the four interviewed teachers, S1T3 and S1T4, claimed that they do not have time to teach critical thinking because they have to cover the content, "finish the program" as they put it. It is worthwhile to mention that both teachers prepare their students for the Lebanese official exams. On the other hand, S1T1 mentioned that content coverage and time restriction should not be considered excuses for not fostering students' critical thinking, because teachers teaching classes that sit for the official exams often finish their programs early and if they plan properly they can allocate time to teaching critical thinking.

Large number of students hinders critical thinking. The principal commented that despite evidence-based data to the contrary, teachers often argued that class size affects their performance and hinders development of critical thinking. Two of the four teachers, S1T2 and S1T4, considered class size an important factor in fostering students' critical thinking. They argued that overcrowded classes and large number of students (30 students per class) were obstacles to teaching critical thinking. S1T2 commented that "With a large class it is difficult to target each and every student to teach critical thinking". The classroom observations revealed that the teachers often could not engage all the students in the lesson. For example, in the classrooms of S1T2 and S1T3, a few students put their heads on the desk to sleep, others did not participate.

Case two.

Principal's, HOD's, and teachers' definition of critical thinking.

Critical thinking involves analysis, reasoning, and logical thinking. Both the principal and the head of the mathematics department agreed that the processes of critical thinking include inquiry, argumentation, objectivity, and deep analysis. The principal stated that critical thinking includes an objective approach and deep analysis. S2H stated in the questionnaire that critical thinking includes "evidence-based reasoning… the ability not to accept anything as an absolute truth, rather to question everything".

The teacher in the written questionnaire gave a comprehensive definition of critical thinking referring to its processes and skills. According to S2T1, critical thinking involves analysis and logical reasoning. In the questionnaire, she defined critical thinking as "Critical thinking is to analyze and to evaluate an issue objectively in order to form a judgment. Critical thinking skills include analyzing, discriminating, comparing answers and situations, logical reasoning, transforming previously learnt knowledge into application". During the

interview, she showed a good understanding of what critical thinking entails in terms of cognitive abilities and skills, but her main focus was on the role of evaluation in critical thinking.

Critical thinking involves making decisions and judgments based on criteria. The principal and the head of the department both agree that critical thinking involves constant questioning and looking for justifications before making a decision or finding a solution. The principal, in the questionnaire, defined critical thinking as "knowing the pros and cons of a situation, having different perspectives, and being able to make judgment to reach a conclusion."

Critical thinking involves problem solving. The principal believed that problem solving is an integral part of critical thinking. According to her answer in the questionnaire, "Critical thinking is a way of thinking to find a solution to a problem". S2H and S2T1 associated critical thinking to problem solving in the context of solving authentic mathematics problems that relate mathematical concepts to real life problems.

Dispositions of critical thinking. S2T1 stated that critical thinking in addition to the students' cognitive abilities also includes students' attitudes towards thinking and education. She stated, during the interview, "A critical thinker has a positive attitude towards thinking, critical thinking, and education". The teacher did not mention any other dispositions of critical thinking.

Critical thinking is a continuous process. S2T1 stated that critical thinking takes time and is a continuous process. During the interview, she added that "Critical thinking is a way of thinking; it is not limited by time. It becomes part of your life and you use it to make decisions in all situations and at all times". The teacher demonstrated a broad understanding of the conditions of critical thinking. Moreover, in the questionnaire, she elaborated that

critical thinking cannot be taught in a few courses at the university level but should be taught at an early age so that it becomes part of the students' thinking style. She concluded the interview by saying that "Critical thinking is not something that you learn, but it is a way of thinking that you get used to".

Teachers' formal education about critical thinking. According to the principal and the HOD, the teachers have acquired their knowledge of critical thinking by attending workshops or seminars organized by different universities. However, both of them could not recall a workshop that was about critical thinking but assumed that the workshops that teachers usually attend include references to critical thinking. Moreover, in spite of the fact that the school has not organized any professional development about critical thinking, the principal expressed her confidence that all teachers were well informed about critical thinking, adopted it, and fostered their students' critical thinking, because critical thinking is a fundamental educational goal in this school and a part of its culture. On the other hand, the HOD during the interview expressed his belief that probably most teachers are not able to give a formal definition of critical thinking, but they "understand what I am saying when I discuss the skills of critical thinking". The head of the department and the teacher both stated that they acquired their knowledge about critical thinking in the university while taking their master's courses. Both of them are currently graduate students majoring in Educational Administration (S2T1) and Mathematics Education (S2H).

Teachers' views on critical thinking and learning mathematics.

Mathematics teachers play a role in enhancing students' critical thinking. The teacher believed that students need the teacher's guidance to develop their critical thinking, but this should be done indirectly. According to her, since in the Lebanese curriculum little space is given to examples and problems that encourage critical thinking and student are not used to thinking critically, then the teacher has an important role in guiding the students,

working in groups, and asking to explain the way they thought to find the answers. She concluded by saying that "Critical thinking is not something that you learn, but it is a way of thinking that you get used to". The teacher says that teaching for critical thinking will help students perform better in mathematics, because they would go beyond application of formulas, computations, and calculations to the next level of understanding mathematical concept and their application in real life.

On the other hand, she expressed the belief that mathematics can develop students' critical thinking by relating it to everyday life situations and having a positive attitude towards mathematics. According to her, mathematics is more than rote memorization of formulas and definitions. However, even though she appreciated the importance of developing students' critical thinking, she stated that she does not plan to teach critical thinking, but she takes advantage of situations or students' questions on the spot to create and discuss problems related to real life and solve open ended questions.

Conceptual understanding of mathematics enhances students' critical thinking. S2T1 highlighted the role of conceptual understanding of mathematics in fostering students' critical thinking. She said during the interview that "They do not just calculate the answer, but evaluate it objectively, understand what it means to find this answer". In the questionnaire, she explained that if we want students to think critically, then the focus should not be the answer, but the analysis and justification of that answer. According to her, a critical thinker is able to explain how he analyzed the situation or the problem and used what he has learnt to solve the problem. During the interview, S2T1 pointed out that the answer is not the main focus, because by applying the formulas and "getting ready for the test", the students can calculate the correct answer and get a good grade without necessarily being a critical thinker. She gave the example of statistics and how students should be able to go further than the mere calculation of the correlation coefficient, to objectively analyze the given factors and

the meaning of that coefficient in relation to the given situation; this requires critical thinking. In the questionnaire, she expressed the belief that "teaching for critical thinking will show students that mathematics is not just application of formulas or computations, but will help them understand the concepts". She associated understanding mathematical concepts with critical thinking in contrast to performing calculations and finding answers.

Teachers' views on instructional strategies that foster critical thinking.

Classroom discussions and asking leading questions foster critical thinking. S2T1 stated that having discussions with the students encourages them to think critically. In the questionnaire, she stated that one of the strategies she uses is "Asking students to justify their answer and explain how they found that answer" as well as working in groups. During the interview, she explained "I ask questions to guide the students; I do not tell them that an answer is right or wrong, but I question their choices and ask leading questions until they discover the correct answer. We build the chapter together". S2T1 was the only teacher that considered asking open ended questions and giving students the chance to analyze and discuss their friends' answers an instructional strategy that encourages students to think critically. The observation of her classroom confirmed that she encouraged dialogue between students and the students showed enthusiasm to participate in the discussions. S2H in the questionnaire stated that as head of mathematics department he encouraged teachers to engage students in discussions as well as dialogue between students to enhance their critical thinking. The classroom observation of S2H also showed that he encouraged students to discuss with each other how to solve a given problem with minimum interference from him. S2H also avoided stating whether an answer is right or wrong and encouraged the students to persist until they reached the right answer. On the other hand, S2H used every opportunity to address students' misconceptions and tried to build on them.

Asking challenging questions fosters critical thinking. S2T1 believed that asking students open-ended questions that have the possibility of different answers encourages them to think critically.

Solving authentic problems fosters critical thinking. S2T1, in the questionnaire, stressed the importance of relating mathematical concepts to real everyday life situations and "analyzing, comparing, being able to know the real meaning of their studies". She said that in all the grades we can relate every concept that students learn to their everyday life; so that they realize that what they are learning in the mathematics classroom is relevant to their everyday life. The teacher gave examples of how she related the concept of proportionality to school tuition raises and discussed the increase of rates. She said that she makes use of situations that happen in the classroom to try and relate it to what they have learnt or are currently learning; this approach, according to her, fosters critical thinking. In the questionnaire, she stated that she incorporates critical thinking in her everyday mathematics teaching by relating what students learn in the classroom to their everyday life.

Teachers' views on barriers to critical thinking.

Students' lack of motivation and reluctance to think hinder development of critical thinking. S2T1 commented that one of the barriers to critical thinking is the students' reluctance to learn, to think, and to put an effort to understand. She explained that students were sometimes not motivated to think and "they would say just give us the formulas". Another obstacle, according to S2T1, is that the students do not have a positive attitude towards learning in general, and mathematics in particular.

Lebanese mathematics curriculum does not foster critical thinking. S2H and S2T1 both agreed that the Lebanese curriculum does not foster students' critical thinking, but stipulated that developing students' critical thinking does not depend only on the curriculum,

but also on the teaching methods: for example, according to them, student centered approach or questioning the truth of everything foster critical thinking. However, S2T1 expressed her concern that the kinds of questions that are asked in the official exams do not encourage students to think critically. According to S2T1, the questions of the official exams are quite similar every year and students do not need to master a subject to pass the exam, all they have to do is revise previous exams and study. The exams, according to her, are not assessing students' knowledge of the material or ability to solve problems; they are assessing the students' ability to study by heart and recall information. S2H also shared this concern when he stated during the interview that in the official exams if you mastered the skill without understanding the concept, then you can achieve good results.

The urgency to cover the content hinders the development of critical thinking. S2T1 believes that content coverage is an obstacle in fostering students' critical thinking. She stated in the questionnaire that "The program, that you have to complete within a limited period of time, does not allow teachers to go deeper in critical thinking, especially in classes that have official exams. You have to prepare your students for the exam and this approach does not foster critical thinking". S2H also shared this concern and explained during the interview "The only obstacle sometimes is time, especially since I teach classes that sit for the official exams". S2T1 suggested that the duration of each session, 45 minutes, is very short and often deep discussions are interrupted; this does not foster critical thinking.

Case three.

Principal's, HOD's, and teachers' definition of critical thinking.

Critical thinking involves analysis, reasoning, and logical thinking. The three teachers stated that critical thinking involves analysis, reasoning, or logical thinking. Two of the three teachers who completed the questionnaires, S3T1 and S3T2, associated critical

thinking with the ability to look at an issue from different angles or solve a problem using different methods. In the questionnaire, S3T1 defined critical thinking as "thinking outside the box", higher level thinking, and added that "logic of higher level requires high level of critical thinking". S3T3 stated that critical thinking involved questioning everything including the status quo, not accepting things at face value, and evidence-based reasoning. On the other hand, the head of the mathematics department, S3H, related critical thinking to logic and reasonable thinking, which is not based on memorization. The head of the department made no references to either the cognitive abilities or the processes of critical thinking; his main focus was logical thinking and analysis.

Critical thinking involves making decisions and judgments based on criteria. The principal, S3P, gave a formal definition of critical thinking making references to the processes of critical thinking as well as cognitive abilities. The definition provided in the questionnaire is the statement about critical thinking presented by Scriven and Paul to the 8th annual conference of critical thinking. One of the three teachers, S3T2, stated that critical thinking includes making decisions based on criteria.

Critical thinking involves problem solving. The teachers associated critical thinking with solving problems that are hard, outside the box, not directly explained in class, and of higher-level thinking. S3T2 during the interview stated that "solving problems will help them think critically". S3H related critical thinking to solving problems that are not classical and the student has to figure out the answer independently. He claimed that critical thinking should be taught at a very early age and all your life. According to him, a student who thinks critically should be able to evaluate whether an answer is logical and make a decision to either refute or accept the answer.

Dispositions of critical thinking. The teachers described a critical thinker as a person who questions everything, a sharp thinker, and confident. According to S3H, a critical thinker is a hard worker, serious, inquisitive, humble, and knowledge seeker.

Teachers' formal education about critical thinking. The head of the mathematics department, S3H, stated that all the teachers know about critical thinking and have participated in conferences and workshops about critical thinking. Two of the three teachers, S3T1 and S3T3, stated that they acquired their knowledge of critical thinking at the university during their masters courses in mathematics education. The third teacher has not received any training or taken courses in critical thinking. Two of the three teachers, S3T1 and S3T2, stated that critical thinking was discussed in some of the workshops they attended in local universities, but none of the teachers have received training or engaged in professional development about teaching for critical thinking.

Teachers' views on critical thinking and learning mathematics.

Mathematics teachers play a role in enhancing students' critical thinking. The three teachers agreed that the students need the teachers' guidance to develop their critical thinking. But they had opposing views on how teachers can guide students to think critically. S3T2 suggested "modeling the way of thinking" and exposing students to and giving them freedom to choose from different methods of solving problems. S3T1 concentrated on giving students time to figure out the solution by themselves by using critical thinking, whereas S3T3 stated that "teachers have to base their instruction on inquiry" and students need the teachers' guidance in acquiring the habit of questioning all the methods they are working with and all the mathematics information they are given.

All the teachers agreed that it is possible to develop students' critical thinking while teaching mathematics. One teacher, S3T1, referred to the nature of mathematics by stating in

the questionnaire that "historically mathematics means learning logic". S3T3 during the interview claimed that "teaching mathematics authentically requires the development of the students' critical thinking; otherwise it would be teaching the applications of mathematics rather than mathematics itself". But two teachers, S3T1 and S3T2, argued that students' background and character also played an important role in developing their critical thinking.

Conceptual understanding of mathematics enhances students' critical thinking. Two of the three teachers, S3T2 and S3T3, during the interviews, stated that the students' critical thinking is enhanced if teachers aim for conceptual understanding of mathematics rather than procedural mathematics. S3T3 in the questionnaire claimed that "In my classes students prefer to do mathematics automatically. They think that they only need the skills and the methods, this opposes the idea of critical thinking". Furthermore, he stated that the Lebanese mathematics curriculum does not aim for conceptual understanding and critical thinking. S3T3 added that mathematics is a domain where knowledge should be reached constructively. S3T2 during the interview stated that "in the Lebanese curriculum, we teach skills, the step that they should follow to find the answer, but we ignore the essence of the problem" and this approach, according to him, does not foster students' critical thinking.

Teachers' views on instructional strategies that foster critical thinking.

Classroom discussions and asking leading questions foster critical thinking. The three teachers agreed that having discussions with the students in the classroom fosters students' critical thinking. The three teachers described the role of the teacher as a person who "tries to develop the discussion rather than give the final answer", asks questions and demands justifications, so that eventually "we can reach together the required concepts". One teacher, S3T1, in the questionnaire mentioned active learning "at the beginning of a new chapter where the students can use their critical thinking so we can reach the required

concepts" and pair or group work; the other two teachers described only whole class discussions. S3T3 during the interview said that "the teacher tries to develop the discussion rather than give the final answer and the students reach the answer by themselves". According to him, this can be achieved by asking questions, asking for justification, evidence, and dialogue between students.

Asking challenging questions fosters critical thinking. Two of the three participating teachers believed that asking students challenging questions fosters critical thinking. S3T1 said that asking students questions that require higher level thinking as synthesis or evaluation on Bloom's taxonomy and finding out what went wrong while solving them, enhances students' critical thinking. S3T1 stated that critical thinkers can solve parts in a test that are outside the box and not directly explained in class; he explained that these parts are added to the test items to make it hard to earn a full grade and if students solve these parts, then they are critical thinkers. S3T2, during the interview, explained that "when I am solving different exercises, I ask them questions that will help them to think critically" and also made a reference to the hierarchy in Bloom's taxonomy. S3T3 said that "I try to develop items that are on higher level of thinking and require critical thinking".

Solving authentic problems fosters critical thinking. The three teachers addressed the absence of real-life authentic problems in the Lebanese curriculum. Two teachers pointed out the lack of real-life authentic problems that make mathematics abstract, while the third teacher mentioned the absence of student-centered projects. S3T1 stated that the curriculum lacks real life applications and the teacher must discuss the relation between math and real life.

Teachers' views on barriers to critical thinking.

Students' lack of motivation and reluctance to think hinder development of critical thinking. The three teachers agreed that student characteristics are one of the challenges they face in developing students' critical thinking. According to S3T3, the students prefer to have the final answer directly without putting an effort in understanding the process and thinking about it, "they ask if it is enough to study the final answer". Another challenge is students' weak academic backgrounds (S3T1) and home environment (S3T2).

Lebanese mathematics curriculum does not foster critical thinking. The three teachers agreed that the Lebanese curriculum is very long and due to time restrictions and official exam preparations, the teachers cannot allocate time to develop students' critical thinking. They described the curriculum as dense, bulky, and full of information and methods that can be memorized; it is based on the skills the students need to solve problems. According to the teachers, these factors do not foster students' critical thinking. Two teachers pointed out the lack of real-life authentic problems that make mathematics abstract, while the third teacher mentioned the absence of student-centered projects. Two teachers, S3T2 and S3T3, during the interviews, stated that the absence of testing for critical thinking in Lebanese official exams did not encourage teachers to teach for critical thinking. S3T3 tried to explain this by the kind of questions that are asked in official exams; questions that students can solve by just mastering the skills without understanding the concept or thinking critically. S3T2 had a similar opinion about official exams and claimed that the curriculum itself develops critical thinking, but the questions in the official exam are classical and quite similar every year. S3T3 reflected in the questionnaire that in spite of the shortcomings of the Lebanese curriculum, teachers can still teach critical thinking, because "CT is a method of instruction and curricula are backgrounds and contexts for this instruction". However, he considered teachers' resistance to teach critical thinking a greater obstacle than the curriculum itself.

S3H firmly believed that the Lebanese curriculum is almost perfect, better than others, and requires students to think logically. However, he reflected that recently the official exam questions are easier, straight-forward, and follow certain steps to reach the solution, only one or two parts require higher level thinking and logic.

The urgency to cover the content hinders the development of critical thinking. The three teachers agreed that the Lebanese curriculum is very long and, due to time restrictions and official exam preparations, the teachers cannot allocate time to develop students' critical thinking. They described the curriculum as dense, bulky, and full of information and methods that can be memorized; it is based on the skills students need to solve problems. According to the teachers, these factors do not foster students' critical thinking. S3T1 in the questionnaire stated that "There is not a lot of time for critical thinking…we have to finish the program and prepare the students for official exams". S3T2 agreed and stated in the questionnaire that "to develop critical thinking you need time, but the curriculum is long and administration sets strict outlines, requires lesson plans, and number of chapters".

Case four.

Principal's, HOD's, and teachers' definition of critical thinking.

Critical thinking involves analysis, reasoning, and logical thinking. S4T1 identified some processes of critical thinking like analysis, interpretation, and reasoning. S4T1 associated critical thinking with higher level thinking.

Critical thinking involves making decisions and judgments based on criteria. S4P defined critical thinking as assimilating the data, bringing it together, comparing and questioning the data, discussing and drawing conclusion. He said that "We should not take reality at face value; we should critique it". According to him, analyzing the situation before

making decisions is a precondition to critical thinking. However, he also expressed the opinion that to be able to think critically, students need a strong knowledge base; "If we want critical thinking to be meaningful instead of just a performance, then the students must have a strong knowledge base to build on". The head of the mathematics department's definition of critical thinking was formal and emphasized the evaluation of sources to make decisions or judgments. S4T1 stated that critical thinking includes decision making. He made no references to criteria or validation of sources.

Critical thinking involves problem solving. S4T1 defined critical thinking as problem solving and thinking "outside the box".

Dispositions of critical thinking. The principal, S4P, described critical thinkers as people that have the ability to think and communicate their ideas, are outgoing, broad-minded, and question and critique everything. No reference was made about other dispositions of critical thinkers. S4H and S4T1 did not discuss the dispositions of critical thinkers.

Teachers' formal education about critical thinking. According to the principal, the teachers that have graduated from universities during the last decade have a good understanding of critical thinking, but for the more experienced teachers the concept is alien. The school has invited guest speakers on two occasions to present themes related to critical thinking, and teachers have attended a workshop with the same topic. The principal thinks that this is not enough, but "we have made an effort". During the interview, S4H expressed his belief that "the teachers teach critical thinking, but they do not know that this is critical thinking". S4T1 stated in the questionnaire that he acquired his knowledge of critical thinking from academic workshops and off-field experience.

Teachers' views on critical thinking and learning mathematics.

Mathematics teachers play a role in enhancing students' critical thinking. S4T1 expressed his belief that the teacher plays an important role in helping his students think critically by encouraging and motivating them. He added that teachers should be "reflective models" for their students, because when the teacher thinks critically and creatively the students will follow by example. On the other hand, he claimed that critical thinking enhances the students' ability to analyze and interpret any subject in a better way; it helps students understand the meaning and relevance of any subject they study.

Conceptual understanding of mathematics enhances students' critical thinking. S4T1 reflected on the relation between conceptual understanding of mathematics and enhancing students' critical thinking. He stated "Mathematics is not about algorithms and properties. It is about understanding a concept. Mathematics requires critical thinking, not only memorization without logic which leads to failure".

Teachers' views on instructional strategies that foster critical thinking.

Classroom discussions and asking leading questions foster critical thinking. One of the instructional strategies that S4T1 mentioned was asking students meaningful and guiding questions to reach a conclusion or understand a concept. He referred to whole class discussion with the teacher guiding the students to the correct answer. There was no mention of student-student dialogues or the possibility of different solutions to open-ended questions, but different ways of finding the same answer.

Asking challenging questions fosters critical thinking. S4T1 stated that challenging question like critical riddles grab the students' attention and develops their critical thinking.

Solving authentic problems fosters critical thinking. According to the S4T1, to be able to think critically, the students should be exposed to real life problems and authentic instruction is essential. He claimed that he introduced each new concept with a real-life problem; "At the beginning of each chapter, I try to give a real-life example regarding that chapter before taking the properties or theorems". But the example that the teacher presented was a problem that had a unique solution with no alternatives, in a real-life context not relevant to the students.

Teachers' views on barriers to critical thinking.

Students' lack of motivation and reluctance to think hinder development of critical thinking. S4T1 expressed concern that some of the barriers to critical thinking were; students' lack of motivation to think on one hand, and the lack of confidence to express their ideas on the other. "Some students are shy; they lack confidence in sharing ideas".

Lebanese mathematics curriculum does not foster critical thinking. According to the teacher, the Lebanese mathematics curriculum does not foster critical thinking, because the topics are explained at face value and are not studied deeply. Moreover, the curriculum does not relate mathematical concepts to real life situations so that the students transfer their knowledge to their everyday life. S4T1 stated that "in Lebanon, most of mathematics, is explained straight forward, without explaining to students where these concepts are used in real life".

The urgency to cover the content hinders the development of critical thinking. Both the teacher, S4T1, and the head of the mathematics department, S4H, agreed that the mathematics program was long and content coverage required more time than was allocated to mathematics lessons. Hence, their urgency to cover the content did not allow them to spend time on solving exercises that foster students' critical thinking.

Comparison among the Views of the Principals, HODs, and Teachers of the Four Schools about the Role of Critical Thinking in Mathematics Classrooms

In this section, I describe the similarities and differences among the views of principals, heads of mathematics departments, and mathematics teachers of the four schools about the role of critical thinking in mathematics classrooms. After data analysis, 16 patterns were identified.

Principals assumed that teachers know about critical thinking and teach for critical thinking even though most of them have not received any training or professional development in critical thinking (P_1). The principals and the HODs of the four schools claimed that the teachers have basic knowledge and teach for critical thinking even if they cannot articulate a formal definition of critical thinking. However, the questionnaires, interviews, and class observations revealed that most teachers did not teach for critical thinking instead they focused on content coverage and preparation for official exams.

Most of the principals, HODs, and teachers lack knowledge in the definition of critical thinking, its skills and dispositions (P_2). The principal of S2 had given a comprehensive definition of critical thinking, whereas the principals of S1 and Four focused on one aspect of critical thinking like justifying your reasoning and drawing conclusions. The HODs of S3 and S4 provided definitions of critical thinking that were narrow in scope. Most teachers' definitions of critical thinking were not comprehensive and included discrepancies. The exceptions were the head of mathematics department and the teacher of S2 as well as one teacher of S3; they stated that their knowledge of critical thinking was based on their graduate studies. It is worthwhile to mention that all participants defined critical thinking in terms of cognitive skills and ignored dispositions of critical thinking.

Teachers lack training and knowledge in instructional strategies that foster critical thinking (P_3). Most teachers stated that student-centered instruction, solving challenging and authentic problems, and relating mathematics to real life foster students' critical thinking. However, the questionnaires, interviews, and classroom observations revealed that most teachers had not received any training to teach for critical thinking, often reverted back to direct instruction, and had limited knowledge of and access to authentic or challenging problems. Two teachers of S1 gave irrelevant examples of authentic problems, one teacher of S2 discussed a relevant authentic problem she has solved in class, the others failed to give any examples.

Principals, HODs, and teachers associated critical thinking with solving challenging problems (P_4). Most of the principals, HODs, and teachers associated critical thinking with solving problems that are challenging and not identical to problems solved in class. They characterized these problems as difficult, out of the box, and requiring higher levels of thinking.

Few teachers believed that students' cognitive abilities play a role in the development of their critical thinking (P_5). The four participating teachers of S1 stated that the teachers' efforts to develop students' critical thinking are limited by the predetermined cognitive abilities of the students. The head of the department and one teacher from S3 stated that students should be taught to think critically at an early age; otherwise, it will not be feasible.

Critical thinking is a continuous process, a way of thinking (P_6). All the participants agreed that critical thinking is a continuous process and a way of thinking that the students acquire and later transfer to their everyday life. They agreed that the skills of critical thinking can be applied in all subject matter and at all times.

Two principals assumed that a strong knowledge base prepares students to think critically (P_7). The principals of S1 and S4 believed that a strong knowledge base is associated with critical thinking; nevertheless, their approach differed from each other. The principal of S1 stated that if students acquired a strong knowledge base, then they will consequently be able to think critically. On the other hand, the principal of S4 stated that a strong knowledge base is a prerequisite of critical thinking; that is, if students did not have a strong knowledge base about a subject area, then they cannot think critically about it.

Conceptual understanding of mathematics enhances students' critical thinking (P_8). Even though the questionnaires and interview questions did not address this issue, most teachers discussed and distinguished between two ways of doing mathematics: procedural and conceptual understanding. Moreover, they associated fostering students' critical thinking with teaching mathematics for conceptual understanding and pointed out that the Lebanese mathematics curriculum and official exams favored procedural mathematics.

Classroom discussions and authentic problems enhance students' critical thinking (P_9). All the HODs and teachers agreed that constructing mathematical knowledge through classroom discussions fosters students' critical thinking. However, their descriptions of classroom discussions and the classroom observations showed that most teachers did not hold classroom discussions that encourage student- student dialogue or divergent thinking. An exception was the head of the department of S2 who both, during the interview and in the classroom, discussed and encouraged student- student dialogues and divergent thinking. The mathematics teacher of S2 gave a relevant example of an authentic problem she has used in the classroom. The classroom observations established that when having classroom discussions, teachers asked low order short questions that encourage convergent thinking.

Teachers do not plan to teach for critical thinking; they do it on the spur of the moment (P_{10}). The answers to a question in the questionnaire that asks to describe a classroom activity that engages students in critical thinking can be summarized as follows: the two participating HODs failed to provide an example, the head of the department of S4 gave an irrelevant example; five teachers did not provide any examples, and the other four gave irrelevant examples. They did not plan to teach for critical thinking. Moreover, two teachers, in S1 and S2 respectively, explicitly stated that they teach for critical thinking on the spur of the moment when opportunities arise in the classroom.

Teachers do not often use the instructional strategies that they believe are conducive to critical thinking (P_{11}). Even though teachers agree that active learning fosters critical thinking, they often revert back to direct instruction and lecturing. When asked to describe a typical lesson most of them listed teaching practices related to direct instruction.

Lebanese textbooks do not include problems that develop students' critical thinking (P_{12}). All the participating teachers agreed that the Lebanese textbooks do not include critical thinking problems. Moreover, the teachers of S1 and S2 justified their claim by making comparisons to American, British, and French textbooks.

Lebanese mathematics curriculum is dense and the urgency to cover content hinders the development of students' critical thinking (P_{13}). All the participants agreed that the Lebanese curriculum is dense, long, and teaches for breadth rather than depth. According to most of them, the teachers' urgency to cover content hinders fostering students' critical thinking especially for classes that will sit for the official exams. One teacher from S1 firmly believed that content coverage does not hinder teaching for critical thinking if the teachers were willing and able to teach for critical thinking. The head of the department and

the teacher of S2 also shared this sentiment because according to them, teaching for critical thinking is teaching a way of thinking rather than content.

Official exam questions are low order questions and similar to questions of previous exams (P_{14}). Teachers that teach classes that sit for the official exams stated that mathematics questions included in the official exams were standard, based on procedures, and similar to questions asked in previous years. According to them, this encouraged teachers to teach for the test. This also became apparent during classroom observations when teachers repeatedly made references to official exams during the lesson.

Teachers do not have time to teach for critical thinking in classes that sit for the official exams (P_{15}). Teachers that teach classes that sit for the official exam stated that preparing students for the official exam did not leave them time to teach for critical thinking. However, one teacher from S1 and the head of the department of S2 disagreed. S1T1 justified her belief by stating that teachers that prepare students for official exams always finish the program beforehand and reserve the last weeks for revision; hence if they planned properly, they can allocate time for developing their students' critical thinking. S2H stated that he taught classes that sat for the official exam, but he did not change his way of teaching that encouraged critical thinking.

Large number of students in a classroom hinders developing students' critical thinking (P_{16}). Three of the four participating teachers of S1 indicated that the large number of students in their classes (28 to 33 students per class) hindered their attempts to teach for critical thinking. On the other hand, the principal of S1 stated that teachers hold the false belief that large number of students in the classroom affects the quality of their teaching negatively. This issue was not mentioned in the other three schools where the number of students in each class varied between 14 and 24 students per class.

School Culture and Critical Thinking

Case one.

Principal's and HOD's views of their school culture. The head of the mathematics department declined to answer the questionnaire or be interviewed, because she stated that her responsibilities and job description do not entail making decisions related to school culture. According to her, she is simply a moderator, a senior teacher, and a person that makes sure that everything runs smoothly in the department. She said that all decisions are made only by the principal

School mission. According to the principal, the school mission fosters students' critical thinking, because it instills in them a sense of responsibility and accountability for their actions. S1P stated that "the students should be able to make responsible decisions given the basic rules; this is critical thinking". A review of the official school mission in the school website reveals that special emphasis is placed on instilling moral and spiritual values in the students and there is no reference to developing students' critical thinking.

The principal ascertained that there are certain aspects of the school culture that encourage students to think critically and mentions as an example the morning chapels. Every morning the student body is assembled in the chapel to pray. During these meetings, guest speakers present different topics, mostly from the bible, but quite often they discuss social issues as well. The principal states that in chapel the students are encouraged to think, "You need to think things through, whether related to the Bible or school affairs". The principal said that he believes that the discussions that take place in the chapel "encourage students to think about what happened, where we are now, and where we are going".

Collaborative leadership. During the interview, the principal discussed how he makes decisions regarding school regulations and described the steps he takes to implement it. He

explained that he took decisions unilaterally, sometimes discussed its ramifications with 3 or 4 teachers, and then asked the teachers to implement it. The teachers did not participate in decision making. Before implementing a new regulation, he considered how it will affect the students and parents and whether it will appease parents' complaints, but he did not discuss how teachers will be affected or how they will react. During the interview, he stated "I asked the teachers to form pairs and then just implemented it". The principal holds faculty meetings twice a year to discuss academic issues and school regulations.

During the interview, the principal stated that he usually resolves disputes between teachers about teaching practices by ignoring them, because he does not like confrontations. But in case the problem persists, then he makes sure to listen to both parties and asks for explanations and reasons for the opposing decisions. The principal insisted that he would fight for the teacher that gives a reasonable justification. Disputes between teachers are mostly referred to the principal.

Teacher collaboration. The principal did not know exactly how many hours teachers spend on dialogue among peers, planning, and working together to evaluate programs. The mathematics, Arabic, and English subjects have subject coordinators that meet with the teachers weekly or every other week. However, the principal pointed out that he prefers the idea of teacher- teacher dialogues instead of teacher-coordinator interactions. He thinks that subject coordinators "wreck teacher- teacher coordination, particularly across subjects", and adds that he assigned coordinators only because the school board requested it. He explains that the formal meetings between teachers and coordinators facilitates vertical coordination, but somehow hinders horizontal coordination across subjects. The head of the mathematics department refused to participate in the study, because according to her, she does not participate in decision making and has no influence on shaping the school culture; only the principal has the right to make decisions.

The principal says that the teachers of the senior department were very friendly with each other till 5 years ago when due to union elections they were divided into different groups. However, he believes that recently the problem has been resolved and relationships have mended. "I think generally it is a good and friendly atmosphere in the senior staff room", but he speculated that "it does not take much to split them". The principal did not talk about the atmosphere in the junior staff room, because there are many teachers, about 150, and the junior department is situated in a different building and it is difficult to understand the dynamic of that group.

Professional development. S1P expressed his concern that he does not allocate enough time for professional development, because teachers usually do not like to stay after school and that is the only time he is free. The principal picks the topics of professional development workshops based on what he thinks that the teachers need. For the last few years, most of the professional development trainings were about basic teaching that can be applied to all levels from KG till grade 12. A few examples are teacher behavior, communication with students, assessment of students' understanding of a delivered lesson, and metacognition. However, the principal was disappointed because even though teachers often approached him to discuss their personal needs, they rarely discussed academic difficulties or asked help for improving their teaching. The teachers have not participated in conferences or workshops about critical thinking.

Principal's and HOD's views on students' participation in school life.

Students are involved in making decisions related to academics or administration. The principal stated that students do not have the freedom to make decisions about academics in the Lebanese baccalaureate. The Lebanese system is restricted; the students are not allowed to choose which section, scientific or humanities, they will go to in the 11^{th} or 12^{th} grade. The decision is made based on their academic ability and grades. According to S1P,

this is related to how society thinks; "the Lebanese society is very afraid of failing". He expressed the belief that students should be allowed to make their decisions and then bear the consequences. However, the A-level (British system) students make decisions and have the freedom to make choices related to academics; which courses to take, which subjects to pursue.

With respect to administrative decisions, the principal commented that students should have an input in school policy, but not a decision-making input. He stated that "school is a dictatorship, not a democracy and I think that is so important. I do not say students should not have an input, but they cannot have a decision-making input". The principal said that students can make suggestions, discuss school regulations with him, and they might convince him to change certain regulations. Furthermore, he explained that he does not allow students to make school policy but allows them to comment on school policy and have a strong input. He demonstrated this point by an example; lately the principal, based on the teachers' suggestions, wanted to implement a new policy for graduation. "I went to the 12th grade and presented the idea to them, we had a 25 minute discussion and I made changes in my decision". The school does not have a student council, but the student body is represented by the student affairs organization that is responsible for social activities in the school.

Students are involved in making decisions related to their social activities in school. The student affairs organization (SAO) organizes all the social activities of the students like bake sales, parties, non-uniform days, and charity contributions; it takes care of what the students want. Its members are students from the secondary classes (10, 11, and 12) that are elected by the students of grades 6 to 12. According to the principal, the SAO used to be an important part of school life, "used to be the highlight 10 years ago", but lately it has lost its appeal since most of the students' social life is associated to social media. The principal

added that he involves students in leadership roles and gave the example of organizing all the details related to participation in Model UN, a project organized by a Lebanese university. The students plan and organize everything; they are completely in charge of conducting meetings, running the extra sessions, and transportation.

Principal's and HOD's views on controversial debates.

Political debates. The principal said that he strongly discourages political debates between students in the school premises, even among teachers. He argued that "Lebanese students naturally cannot discuss, cannot listen to the other person's opinion, they can only shout and people get hurt. It is not a discussion, just a shouting match". Hence to avoid confrontations and fights, teachers and students are discouraged to participate in political debates.

Social debates. The students have a platform to discuss controversial social issues during the general studies hour (one hour per week) for secondary classes (10, 11, and 12), and in this period "they can discuss anything they want". But according to the principal, the last time he recalls the students discussing an important controversial social issue was 7 years ago.

Principal's and HOD's views on the epistemological orientation of the school. The principal stated that the dominant mode of instruction in the school is the traditional mode of instruction, when the teacher as "the cleverest person in the classroom" tells the students what they will discuss in class, delivers the lesson, and the students are free to ask questions and they will be tested later. The principal said that he believes that this is how it should be instead of forming random groups and asking students to discuss things they absolutely have no knowledge about. One of the interviewed teachers expressed a similar idea when he described a typical lesson as the teacher delivering a lecture, stating and explaining the

properties, and solving exercises to reinforce students' knowledge. The classroom observations corroborated this approach; the teachers lead the classroom instruction by reminding students the main points of the chapter and later asking low level questions based on recall and application.

Case two.

Principal's and HOD's views of their school culture.

School mission. Both the principal and HOD agreed that the school mission fosters students' critical thinking, because it is based on the principals of free expression of ideas and opinions, questioning the status quo, and developing students' leadership skills. The principal, S2P, focused on the student- teacher- administration relationship and the importance of encouraging students to express their views and ask for justifications. During the interview, she said that "the students always have the right to ask why, how did you make this decision, and thus take part in decision making. This is how we include critical thinking in our school culture". On the other hand, the HOD, who is a former student of the school, stated that the school mission is to prepare students capable of questioning the status quo and assuming leadership roles in the community, and to be able to achieve that, students have to think critically. A review of the official mission statement of the school revealed that critical thinking and the dispositions of a critical thinker are explicitly included in the objectives of the school. The principal explained during the interview that "we try to develop three traits in our students: mutual respect, self-regulation, and listening to each other".

The principal and the HOD indicated that the factors that foster students' critical thinking are: exposure to critical thinking at an early age and in all subjects, respect and positive attitude towards students and their thoughts, free expression of ideas, and teachers and administrators as role models. They said that they have an important role in fostering

students' critical thinking, but they envisaged their roles in different domains. The principal put the emphasis on her style of communication with the students that demonstrates critical thinking and encourages students to adopt this way of thinking, whereas the HOD emphasized his responsibilities in the development of curriculum and instructional strategies that enhance students' critical thinking.

Collaborative leadership. The principal, S2P, and the head of mathematics department, S2H, claimed that all disagreements about teaching practices are resolved through dialogue, free exchange of ideas, and giving justifications for the expressed ideas. It is worthwhile to mention that the HOD explained that when he tries to convince the teachers to make a change or apply a new method, and they insist on their position, he allows them, against his better judgment, to continue teaching in the old method and revisits the issue later; that is, he does not enforce his decision but works towards a consensus. He said that "the teacher is the master of the class and knows the needs of his/her students better than me". This attitude is in alignment with the principal's mode of communication with the teachers. The principal states that her style of leadership is participatory; during staff meetings she presents the problem, asks for the teachers' feedback, then they think together, discuss different options, and make a decision. Moreover, she added that "before taking an abrupt decision, we should pass through a process of examining the situation, asking questions like how and why, and taking into consideration different aspects of the problem". She explained that this is her mode of interaction with the students and the teachers. The teachers participate in decision making about most academic issues and some administrative issues as well. However, she added that sometimes the administration makes the decision.

Teacher collaboration. Teachers have weekly meetings with the subject coordinators for planning and evaluation, as well as four meetings per year for each class they teach. In addition to these meetings, heads of divisions, subject coordinators, or the administration may

also call for meetings with specific agendas. The HOD explained that previously the teachers used to cooperate and meet informally, but now communication is more planned and structured. They gave examples of teachers collaborating to address disciplinary problems of a disruptive class or designing a geometric analysis model for grade six to grade nine. The principal and the HOD described the relationship among teachers in positive terms as "harmonious", "professional", "constructive", and based on mutual understanding and respect.

Professional development. The teachers are encouraged to participate in the different workshops that the school receives invitations to. Sometimes the school invites a guest speaker to give workshops or lectures and the topics are chosen by the administration according to teachers' needs. However, the principal and the HOD could not remember whether the teachers have participated in any conferences or workshops about critical thinking. The HOD said that he thinks that he should initiate professional development for critical thinking since some of the teachers do not have a background in education and might not be cognizant of critical thinking.

Principal's and HOD's views on students participation in school life.

Students are involved in making decisions related to academics or administration. The principal said that the school encourages students to take initiatives and participate in school life by organizing events, commemorations, marches, trips, social gatherings, and the school fair. The students have a school council in the secondary classes that is elected by them, one representative from each class. The students sometimes participate in decision making like changing the date of the ski camp or taking reading days before the exams. The principal stated that "we never reject a proposal without consideration". She demonstrated this point by giving an example of how students participate in making school policy; the secondary school students suggested changing exam policies and giving students additional

reading period before the exams, and after deliberation, the policy was changed. The principal explained "I would not say they made an administrative decision, but they made a proposal and it was accepted". Moreover, she added that the kind of uninhibited and open relationship that exists between student-teacher-administration encourages students to think critically.

The students that participate in the French Baccalaureate have the additional responsibility of making decisions related to academics like which language to choose as a first language based on their strengths and weaknesses. According to the principal, this type of deliberation, self-reflection, and decision making, based on criteria, enhances students' critical thinking.

Students are involved in making decisions related to their social activities in school. The student council plans and organizes all the social activities in the secondary school. In their yearly plan, they include visits, lectures, outings, social gatherings, and commemorations (national, cultural, sports). The principal during the interview stated that "the students plan and organize everything; we only inquire why you have chosen this specific topic or why you have decided to visit this site". The students also participate in organizing the yearly school trip and fair.

Principal's and HOD's views on controversial debate.

Political debates. According to the principal, political debates are not held in a planned and pre-determined manner; rather they are initiated by the teachers or students in case of a national crisis or an international incident. The principal explained that students are often misguided by reading articles on social media and the teachers', specifically history teachers', objective analysis and expert opinion about the situation can help them understand better. The head of the department also confirmed that in the school, students are encouraged

to discuss political issues and there are no restrictions on topics, students are allowed "to question everything, to question authority".

Social debates. The school has a social and character development program for grades 1 to 12 that includes presentations and debates about health, sexual education, decision making, and leadership skills. The principal explained that the school encourages these debates, because they provide the students a safe place, in the presence of a professional or a teacher, to hear different and opposing views and learn to respect them. The head of the mathematics department said that mathematics questions that are related to economics often provide an opportunity to discuss social issues in class.

Principal's and HOD's views on the epistemological orientation of the school. The principal and HOD said that the dominant mode of instruction in the school is student centered and based on discussions between teachers and students. Nevertheless, both of them expressed their belief that teachers often revert back to teacher centered, "the classical method of instruction", or direct instruction in some lessons. The HOD explained that this is due to the fact that most of the teachers were themselves taught in the classical method. But the principal related this factor to the teachers' "rush to cover the program" and concluded that this hinders critical thinking.

Case three.

Principal's and HOD's views of their school culture.

School mission. Both the principal and the head of department stated that the school mission fosters students' critical thinking. The principal explained that the school has adopted developing students' critical thinking as one of their goals for the next five years, and the whole school will be mobilized to act accordingly; the administration, heads of divisions, and HODs. In the questionnaire, the principal of the school stated that the school's main goal is "to provide our students with quality education, and equip them with the tools necessary to

overcome the challenges of the 21st century" and considered critical thinking one of the main tools for reaching this goal. According to the principal, the school atmosphere is the main factor that affects students' critical thinking, as well as their family life and choice of friends to a lesser degree. Furthermore, he concluded that this school with its highly professional department heads, qualified and young teachers, and supportive administration and school board creates a culture that fosters students' critical thinking. Moreover, the principal believed that teachers who did not adopt this idea should be replaced.

The head of the department emphasized that the most important thing the school teaches its students is how to think, the way of thinking. "They own the logic", he added during the interview. He demonstrated his point by the academic success of their students in different universities and challenging majors.

Collaborative leadership. The administration in cooperation with the heads of divisions and HODs sets the school policies. In case of disagreements among teachers about teaching practices, a hierarchy is set to resolve the issue: first, the teacher addresses the problem with the head of the department, then the head of the division, and eventually the administration. The teachers participate in decision making related to disciplinary issues. They recently made policy changes addressing disciplinary problems and the teachers presented the problems and the tentative solutions during meetings. According to the head of the department, there are no disagreements among the teachers about their practice because there are no parallel classes with different teachers and teachers usually understand school policies and comply with the expectations.

Teacher collaboration. According to the principal, the teachers allocate a lot of time for dialogue among peers, planning, and evaluation often staying at the staff room after school hours. On the other hand, the head of the department stated that because of his tight schedule he does not hold formal and regular meetings with the teachers, and his

responsibilities mostly include checking the mathematics exams for mistakes and guiding or helping the teachers when needed. The head of the department said that he meets teachers often informally, during recess, and is available to answer their questions or help them; he added "we do not have a fixed timetable for dialogues and coordination". However, he insisted that the teachers were always in touch with him and with each other to share new methods, interesting mathematics problems, and news.

The principal also emphasized this atmosphere of collaboration among teachers and gave the example of how teachers and HODs cooperated to solve pending academic issues after the sudden resignation of the academic director, and the heads of divisions and HODs cooperated with his full support. The principal and the head of the department agreed that the overall atmosphere of the staff is friendly, understanding, positive, and respectful. The principal stated that "At our school, we are proud to consider ourselves members of a big family, rather than just colleagues", and the head of the department declared that "there is love and understanding between us and an atmosphere of sharing". Disagreements are resolved in a way that "enhances trust and boosts fairness".

Professional development. According to the principal and the head of the department, teachers regularly participate in workshops organized by esteemed educational institutions or the school itself. Critical thinking was one of the topics of the workshops organized by the school. The principal believes that "professional development is a key factor in maintaining a high quality faculty, the members of which are up to date with the latest education techniques". The principal and the head of department admitted that the topics are chosen by the administration based on the needs of the teachers and the students.

Principal's and HOD's views on students' in school life.

Students are involved in making decisions related to academics or administration. The principal did not discuss students' involvement in decision making except in the context of participation in school social events.

Students are involved in making decisions related to their social activities in school. According to the principal, the students are encouraged to actively participate in school life and decision making. However, he explained this participation in terms of taking part or helping out during school events, sports tournaments, or planning how to volunteer for events that the administration has decided to partake in.

Principal's and HOD's views on controversial debates.

Political debates. The principal said that the school bylaws forbid political discussions of any kind in the school. The head of the department showed concern that even if given a chance, the students are not interested in political issues, because according to him, this is a generation that "only knows how to have fun and dance".

Social debates. The principal stated that he currently does not encourage debates about controversial social issues. The head of the department commented that students are interested in social issues, but these issues are not discussed during mathematics lessons.

Principal's and HOD's views on the epistemological orientation of the school. The principal and the head of the department did not share the same views about the epistemological orientation of the school. While the principal stated that "The dominant mode of instruction in our school is a mixture of student centered and teacher centered systems", the head of departments' approach was more traditional based on direct instruction and strict discipline; "the students are not allowed to talk to each other, they sit as if they are in church", he stated during the interview. The principal is aware that some subjects are still taught by "traditional lecturing" but stated in the questionnaire that certain classes are student centered and rely on "the interaction of students with each other and with the teacher, critical

thinking, experiments and analysis" as well as the use of technology. The classroom observations revealed that the teachers taught their classes in different ways; one teacher relied on classroom discussion to construct knowledge, while another used the method of drill and practice. However, even during the class discussion, the questions were low order questions.

Case four.

Principal's and HOD's views of their school culture.

School mission. Both the principal and the head of the department agreed that developing students' critical thinking and preparing critical thinkers was an important part of the school mission. According to the principal, the school mission dictates that instruction be based on the principle that students acquire knowledge through research, discussions, and debates; this approach enhances students' critical thinking. Moreover, according to the principal, the development of students' free and strong personalities is a part of the school mission and this goal cannot be achieved without making critical thinking a part of the educational system. However, the principal admitted that not all teachers adopt this approach, implement it, or implement it correctly.

The principal believed that even though he does not have powerful levers to ensure that teachers foster students' critical thinking; nonetheless, he plays an important role by continuously observing, commenting, encouraging, and advising teachers to enhance students' critical thinking. He specifically highlighted the importance of communicating this fact to HODs. The head of the department indicated that one of his responsibilities as the head of the department is to enhance students critical thinking by asking teachers to use strategies as asking the students questions that need thinking, asking the students to seek information, and make judgments. Moreover, the principal claimed that extracurricular activities and the social hour on Fridays give students opportunities to foster their critical

thinking too and his role is to ensure that teachers and students approached these activities from this perspective.

Collaborative leadership. The principal explained that the teachers are encouraged to participate in decision making regarding school policies for class promotion, discipline, or academic issues like developing the scope and sequence of their subjects. He said that he tries to make teacher meetings a platform for critical thinking, where he urges teachers "to go beyond simple analysis, to raise questions, express opinions and views, and think deeply about them".

Teacher collaboration. Teachers and HODs meet on a weekly basis to finalize the lesson plans and discuss the methodology. One yearly staff meeting is held in addition to frequent cycle meetings and subject meetings. The principal believes that they have successfully implemented vertical coordination of subjects but have failed in horizontal coordination across grade levels. The head of the department characterized the mathematics teachers as cooperative and stated that they are always discussing new ideas, experiences, and techniques. However, the head of the department could not provide an instant when teachers worked cooperatively to achieve a common goal.

Furthermore, in an attempt to analyze the relationship among teachers, the principal categorized teachers into three groups. The first group is formed of teachers who are dedicated, diligent, love their job, and open to new ideas. The second group includes teachers who are set in their ways and reluctant to change. The third group is formed of teachers who do not like their job. According to the principal, the relationship between these three groups is not always smooth and supportive, because the innovative teachers would force the other teachers to take initiative as well. The kindergarten teachers have very close relationship as well as the teachers of the same cycle, but the relationships between teachers

of different cycles need improvement. The head of the department stated that the teachers worked as one group and that there was complete trust among them.

Professional development. The principal stated that the teachers participated in workshops organized by the school or the educational council. The topics of the workshop were either suggested by the teachers or the administration. The head of the department said that his role in creating professional development was identifying the needs of the teachers and preparing them for the "staff development event".

Principal's and HOD's views on students' participation in school life.

Students are involved in making decisions related to academics or administration. The principal personally meets with the students of the secondary cycle to discuss the regulations and disciplinary actions. "You tell me this rule is strict, give me an alternative, let us interchange our roles, you are the principal and you face this problem, how would you solve it…make a decision and justify it logically".

Students are involved in making decisions related to their social activities in school. The students participate in school life through the student council that is elected by the students of classes 10, 11, and 12. The student council organizes social events, gatherings, activities. The school also has a committee responsible for publishing the yearbook.

Principal's and HOD's views on controversial debates.

Political debates. The principal expressed his annoyance that students in general were not interested in politics, were not prepared to have political debates, and had a "so what attitude" towards politics. The principal encouraged political debates during civics, sociology, or history lessons. However, the head of the department believed that students are not allowed to discuss controversial political issues.

Social debates. On the other hand, both the principal and the head of the department agreed that the students were interested in social issues, especially related to social media and its effects on the new generation and the platform for these discussions is the social hour on Fridays.

Principal's and HOD's views on the epistemological orientation of the school. The principal stated that students acquire knowledge in the school through a mixture of two methods: transmitting knowledge in teacher centered classroom or cooperatively exploring knowledge in a student-centered classroom. The teachers who use the first method start the lesson by explaining the chapter, giving the students an opportunity to ask questions, and then checking their knowledge. The teachers using the second method start with an open discussion, followed by student conducted research and presentations. The principal explained that the teachers justify the use of direct instruction by time restrictions and necessity of program coverage. The head of the department's views on acquisition of knowledge is based on the following: direct experience, mastering skills, and asking questions. However, the example that he provided was the "direction instruction model" as he called it. The model included explanation of the lesson, guiding the students through the steps, and helping them master solving problems independently. It did not include open discussions, open ended questions, or solving authentic problems.

Comparison among the Views of the Principals and the HODs of the Four Schools about their Role in Creating a School Culture Conducive to Critical Thinking

In some schools, the teachers participate in decision making; in others the administration makes the decisions (P'_1). In this aspect the schools were divided into three types: first, teachers participated in both academic and administrative decision making (S2

and S4); second, the principal made all the decisions by himself (S1); third, there was a bureaucracy that governed decision making (S3).

Principals do not allow students to make academic or administrative decisions but encourage them to have an input (P'_2). All the participating principals stated that students did not participate in decision making in the school. However, the principals of S1, S2, and S3 said that they encourage students to have an input and take their suggestions into consideration when making new school policies or disciplinary rules.

Principals encourage students to take initiatives and make decisions about social activities in school (P'_3). Three of the schools (S1, S2, and S4) have student councils or student committees that organize students' social life in the school. The activities of these councils are limited to organizing outings, parties, social gatherings, and lectures.

Some schools have policies that prohibit students to engage in political debates; others encourage them (P'_4). Two schools (S2 and S4) allow students to participate in political debates about recent international events; these debates are not planned and are held as a reaction to certain events. The other schools categorically prohibit political debates among students and teachers.

Schools encourage students to engage in debates about controversial social issues (P'_5). All the principals admitted that they encouraged debates about controversial social issues. Moreover, three of the four schools had allocated time and platform for these debates indicated as social hour or general studies.

Principals associate developing students' critical thinking with extracurricular activities rather than curricular activities (P'_6). When asked about aspects of their school culture that encourages students to think critically, none of the principals referred to the epistemological orientation of the school or the curriculum. The principals of S1, S2, and S4

referred to extracurricular activities; the principal of S3 indicated the organizational hierarchy and staff.

The epistemological orientation of some schools is characterized by direct instruction and convergent thinking; in others it is based on exploration of knowledge and divergent thinking (P'_7). The principals of S2, S3, and S4 stated that the epistemological orientation of the school was based on the exploration of knowledge and divergent thinking; the principal of S1 stated that the mode of instruction is direct instruction. However, there were discrepancies between the principals' claims, the HODs' claims, and actual practice.

A shared vision of critical thinking as an educational goal contributes to creating a school culture conducive to critical thinking. The mission statement of S2 lists critical thinking as an educational goal; there was alignment between the views of the principal, the head of the mathematics department, and the mathematics teacher about the role of critical thinking in education; they had sufficient knowledge of critical thinking and the teaching practices that were student-centered and based on inquiry fostered students' critical thinking. In the other schools, either they did not adopt critical thinking as an educational goal (S1) or there was no alignment between the views of the participants and the teaching practices.

Emerging Themes

The researcher tried to find the emerging themes by combining the different patterns. Data analysis from the four schools revealed that there are some similarities between the views of principals, heads of mathematics departments, and teachers about critical thinking in mathematics classrooms as well as some differences. There seemed to be differences in the views of principals and heads of mathematics departments about school culture conducive to

critical thinking in terms of shared vision, collaborative leadership, teacher collaboration, and epistemological orientation of the school.

Lebanese mathematics curriculum and official examinations do not foster critical thinking (P_8, P_{12}, P_{13}, P_{14}, and P_{15}). All participants agreed that the Lebanese curriculum, in general, and the mathematics curriculum, in particular, do not foster students' critical thinking. The principals (three out of four) stated that the Lebanese curriculum is dense and focuses on breadth rather than depth of the content; this does not foster students' critical thinking. Eight of the nine participating teachers, and two of the three heads of mathematics departments agreed that content coverage and time restrictions do not encourage teachers to teach for critical thinking especially in classes that sit for the official exams. Seven of the nine participating teachers stated that the Lebanese mathematics textbooks do not include questions that require students to think critically as well as authentic problems that relate mathematical concepts to real life situations. The teachers' views in this aspect were based on their comparisons of Lebanese mathematics textbooks to American, British, and French textbooks. Moreover, according to the teachers, the preparation for the official exams encourages teachers to teach for the test instead of developing students' critical thinking, because of time restrictions as well as the type of questions that are asked in the official exams.

An interesting theme that emerged during data analysis was that teachers differentiated between two ways of doing mathematics: procedural and conceptual understanding. Teachers associated critical thinking with conceptual understanding of mathematics and stated that the Lebanese mathematics curriculum is based on procedural mathematics.

Principals', HODs', and teachers' knowledge of critical thinking and its instructional strategies is limited (P_1, P_2, P_3, P_4, P_5, P_7, P_9). Data analysis revealed that

most of the teachers' knowledge of critical thinking and instructional strategies that foster critical thinking were not comprehensive and included some misconceptions. It is worthwhile to mention that the common factor between the participants who had a comprehensive knowledge of critical thinking and whose teaching practices reflected their knowledge was their educational background and time of graduation; they have either recently received a master's degree in education or are currently graduate students. This is in accordance with the view of the principal of S4 stating that teachers who have graduated in the last ten years have a better understanding of critical thinking and what it entails. Most teachers defined critical thinking in terms of cognitive abilities and ignored dispositions; moreover a few teachers had misconceptions about critical thinking like it being an innate ability or a gift which students either possessed or lacked. Some principals associated a strong knowledge base with the ability of the students to think critically. All the participants agreed that critical thinking is a continuous process, a way of thinking that transfers to students' everyday life.

The instructional strategies that teachers mainly focused on to foster students' critical thinking were classroom discussions and solving authentic problems. However, the classroom observations revealed that not all teachers used these strategies in the classroom. A few teachers and HODs used dialogue among the students to solve mathematics problems; the others mostly used leading questions to guide the students. Most participants associated critical thinking with solving challenging questions that were different and more complicated than the exercises solved in the classroom; thus, associating critical thinking with the ability to solve difficult problems.

A shared vision of critical thinking as an educational goal, teachers' strong knowledge in critical thinking and its instructional strategies, professional development and training in teaching for critical thinking create a school culture conducive to critical

thinking (P'_8, P_2, P_3, P_{10}, P_{11}). The four participating principals expressed their belief that teachers are cognizant in critical thinking, its skills, and instructional strategies. Moreover, even though none of the schools has organized professional development or trainings in teaching for critical thinking, the principals were confident that most teachers taught for critical thinking and fostered their students' critical thinking. The principals of S2, S3, and S4 declared that fostering students' critical thinking was a worthwhile educational goal and one of their school's objectives, whereas the principal of S1 acknowledged that in recent years critical thinking has lost its appeal as an educational goal. The mission statement of S2 listed developing students' critical thinking as an objective of their school; the mission statements of the other schools did not make any reference to critical thinking. Moreover, the principals' information about teachers' knowledge of critical thinking and how they fostered students' critical thinking was sometimes vague and dissimilar to the views of the HODs.

In the absence of a shared vision of fostering students' critical thinking, the schools have not organized any professional development or trainings about critical thinking; hence, the teachers' narrow scope of understanding and misconceptions about critical thinking and how to teach for critical thinking. The teachers participated in different workshops organized by local universities or the school boards, and even though none of the topics were directly related to critical thinking, the principals assumed that they included references to teaching for critical thinking. S4 was the exception since the teachers participated in professional development about critical thinking.

An epistemological orientation based on student-centered exploration of knowledge and divergent thinking, and debating controversial issues create a school culture conducive to critical thinking (P'_4, P'_5, P'_6, and P'7). In terms of the epistemological orientation of the school, S2, S3, and S4 advocated a student-centered and divergent thinking epistemology in contrast to S1 whose principal promoted direct mode of instruction. However, the

three principals who preferred student centered instruction admitted that teachers in their schools often used direct instruction and lecturing. However, there were some discrepancies between the views of the principals and the heads of the mathematics departments in S3 and S4 with respect to the epistemological orientation of the school. The principals claimed that the school's epistemological orientation is based on the exploration of knowledge and divergent thinking, whereas the HODs described teaching practices based on the direct instruction model. Based on the views of the principals and the heads of the mathematics departments of these schools, it can be concluded that the schools strive for an epistemological orientation that is based on the exploration of knowledge, but teachers often revert back to direct instruction and lecturing. According to them, the relapse to direct instruction can be explained by the curriculum requirements and official exams on one hand and the teachers' beliefs, lack of knowledge, and training to foster critical thinking on the other. In S2, both the principal and the head of the mathematics department agreed that the school's epistemological orientation encouraged divergent thinking; however, they both admitted that sometimes experienced teachers persisted in teaching using traditional methods like lecturing and direct instruction.

Debating controversial issues fosters students' critical thinking. It was interesting to notice that when discussing debates about controversial issues some principals differentiated between social and political issues; that is, they encouraged debates about controversial issues and prohibited political debates. The principal and head of department of S2 stated that their school encouraged political and social debates. The principal of S4 stated that he encourages both political and social debates, but the head of mathematics department thought that political debates were prohibited. Both S1 and S3 allowed social debates and prohibited political debates. Moreover, conducting debates about controversial social issues were planned and structured, whereas political debates if allowed took place only in the case of national or international crisis.

A collaborative school culture based on shared vision, collaborative leadership, and teacher collaboration empowers teachers to teach for critical thinking (P'_1, P'_2, P'_3). There were some differences in the leadership styles of the principals of the four schools. The principals of S2 and S4 encouraged teachers to participate in decision making and collaborate with each other; they had regular teacher meetings and subject coordination hours to achieve this goal. However, the head of mathematics department and the teacher in S2 felt more empowered to initiate changes in curriculum and instruction than S4. S3 was characterized by a bureaucratic hierarchy of HODs, division heads, and administration as well as regulations to guide their interactions. The work of the teachers of this school was restricted by a predetermined syllabus, number of chapters, and number of hours allocated to each chapter. Finally, in S1 decision making was central and embodied in one person, the principal; the head of mathematics department of S4 declined to answer any questions about school culture because she stated that she did not participate in decision making, but was a facilitator rather than a head of department. Teachers in S1 did not claim responsibility for their shortcomings in teaching for critical thinking, but put the blame on their students' cognitive abilities, curriculum, time restrictions, and large number of students in classrooms; thus, admitting their inability to instigate change. Leadership style and the extent of teacher collaboration played a role in how empowered the teachers felt to initiate change and improvement.

Chapter Five

Discussion and Conclusion

The purpose of this study was two-fold: first, to study principals', heads of mathematics departments', and mathematics teachers' views of the role of critical thinking in mathematics classrooms; second, to study the views of principals and heads of mathematics departments of their role in creating a school culture conducive to critical thinking. Data analysis led to the emergence of six themes. In what follows, I discuss the themes by relating them to the existing literature, derive conclusions, highlight the contributions made by this study, discuss limitations, and elucidate directions for future research and practice.

Lebanese Mathematics Curriculum and Official Examinations Do Not Foster Critical Thinking

The Lebanese curriculum, in general, and the mathematics curriculum, in particular, as well as the official examinations do not encourage teachers to teach for critical thinking and to foster their students' critical thinking. This result does not reflect the goals and objectives of the Lebanese mathematics curriculum that considers developing students' critical thinking as one of its major goals (CERD, 1999). Additionally, the Lebanese textbooks, in contrast to American, British, and French textbooks, do not include problems that develop students' critical thinking like authentic problems relating mathematical concepts to real life situations relevant to the students. Based on the collected data, the teachers were aware of two ways of teaching mathematics: procedural mathematics and conceptual understanding of mathematics and associated conceptual understanding to the development of critical thinking. On the other hand, procedural mathematics characterized by solving problems using algorithms and prescribed procedures was considered a hindrance to the development of students' critical thinking. However, the Lebanese official exams, in terms of content breadth and the type of questions asked, encouraged teachers to teach for the

test, in other words, to teach procedural mathematics; hence minimizing their efforts to teach for critical thinking. These results are in accordance with the conclusions derived in a report by the ministry of education in Lebanon; the report stated that in general the Lebanese curriculum and teaching practices do not aim to enhance students' critical thinking (Ministry of Education, 2006). Moreover, these results are similar to the results of research conducted in other countries where formal testing resulted in teachers teaching for the test instead of aiming for conceptual understanding or fostering critical thinking (Alazzi, 2008). One aspect that seems unique to the context is that the teachers realized that the Lebanese mathematics textbooks and assessments were not aligned with the goals and objectives of the mathematics curriculum; these results show that the curriculum aims to foster students' critical thinking whereas the textbooks and assessments do not develop students' critical thinking. Further research should be conducted to study the alignment of curricular goals, mathematics textbooks, teaching practices in the mathematics classroom, and official exam questions in the Lebanese context.

An Epistemological Orientation Based on Student-Centered Exploration of Knowledge, and Debating Controversies Create a School Culture Conducive to Critical Thinking

The views of principals, HODs, and mathematics teachers about an epistemological orientation that fosters students' critical thinking can be summarized as: exploration of knowledge, divergent thinking, and utilizing student-centered instructional strategies foster students' critical thinking in contrast to information transmission, convergent thinking, and direct instruction, which do not. Moreover, student-centered instruction and active learning characterized by students conducting research, preparing projects and classroom presentations as well as classroom discussions and student-student dialogue developed students' critical thinking. On the other hand, direct instruction in the form of lecturing and information transmission did not encourage students to think critically. These findings are in

alignment with previous research that investigated instructional strategies that fostered students' critical thinking and school culture conducive to critical thinking (Abrami et al., 2015; Browne & Freeman, 2000; Tsui, 2000).

The principals, HODS, and teachers believed that asking challenging or open-ended questions in new and unfamiliar contexts for the students and solving authentic problems enhanced critical thinking whereas convergent and short questions developed their procedural skills. However, under the pretense of the necessity of content coverage, time restrictions, curriculum requirements, and official examinations, most teachers often reverted back to direct instruction and lecturing. In fact, the absence of planning, support, and commitment to guide the change in teachers' epistemological orientation and the fact that most teachers had been taught by the traditional methods of lecturing and direct instruction resulted in the reluctance of teachers to use student centered instruction, to encourage divergent thinking, and to teach for critical thinking. These findings are aligned with previous research (Gill & Boote, 2012; Gruenert, 2000; Tsui, 2000).

There was disagreement between the principals and the HODs about the epistemological orientation of their schools in terms of desired epistemology versus actual practice. The HODs of the schools were more informed than the principals about the existing discrepancy between declared epistemological orientation and actual practice in the mathematics classrooms. This aspect is a unique finding in the Lebanese context where the principals declare that the epistemological orientation of the school is based on exploration of knowledge and the HODs admit that instruction is based on information transmission and lecturing in the mathematics classrooms.

Another unique aspect of the findings was the principals' attempts to fill the gap that exists in fostering students' critical thinking by focusing their attention on extracurricular activities like the publication of the school yearbook or paper, social hours dedicated to

discussions and debates between students, and the organization of school events and social activities to promote students' critical thinking. The principals encouraged students to participate in school life through student councils elected by them or assigned by the administration. The responsibilities of the student council varied among the schools, but in general included the organization of social events, trips, visits, school fair, lectures, and debates. Some of the schools also published either school yearbooks or periodicals that included students' articles and literary pieces; the publication of the yearbook was considered a great way of developing students' critical thinking, because it gave them the freedom to explore and express their ideas.

Debates about controversial issues foster critical thinking. Review of the relevant literature revealed that to create a school culture that fosters critical thinking, controversial issues should be brought to class and discussed (Browne & Freeman, 2000). The principals, HODs, and teachers agreed that debating controversial issues fostered students' critical thinking. However, an interesting pattern emerged through this research; the principals agreed that students should debate social controversial issues without any reservations but disagreed about discussing political issues at school. Moreover, most schools had platforms for social discussions like an hour per week called the social hour or a program for social and character formation of students.

However, when discussing political debates, they were divided: two principals said that holding political discussions at school was categorically prohibited; the others stated that they allowed political debates to take place occasionally. This aspect of duality in the principals' views about social and political debates is unique to the Lebanese context and can be explained by the precarious political situation in the country. Lebanese citizens in general, and students, in particular, are not capable of discussing political issues; they often get carried away by their beliefs and emotions, tend to shout without listening to the other person's

opinion, and are guided by prejudice and bias instead of facts and logic. However, in my opinion this should give more incentive to debate controversial issues and create a school culture that promotes critical thinking and diminishes bias and prejudice.

It is worthwhile to mention that the two schools which forbade political debates were characterized by a diverse student body (different religious sects and nationalities) or strict norms and regulations, whereas the schools that allowed political debates had a homogeneous school body. The two schools that were lenient in allowing political debates did not plan these debates, but they had them occasionally when national or international crises occurred; in contrast to debates about social issued which were planned and predetermined.

It can be concluded that a school culture that encouraged the exploration of knowledge and divergent thinking, debates about controversial issues, and student participation in school life fosters students' critical thinking.

A Shared Vision of Critical Thinking as an Educational Goal, Teachers' Strong Knowledge in Critical Thinking and Its Instructional Strategies, Professional Development and Training in Teaching for Critical Thinking Create a Culture Conducive to Critical Thinking

The findings revealed that all the principals and HODs agreed that their school mission aims to develop and foster students' critical thinking; however, there were disagreements between them about teachers' knowledge of critical thinking and its instructional strategies as well as whether teachers taught for critical thinking or not. Most of the principals believed that teachers in their schools taught for critical thinking even if their knowledge of critical thinking was not comprehensive, while HODs were more hesitant to admit that teachers taught for critical thinking. The principals' views in this aspect are not in accordance with the relevant literature on two levels: first, to teach for critical thinking, teachers must be cognizant of the skills, dispositions, and instructional strategies of critical

thinking (Innabi & El Sheikh, 2007; Howe, 2004; Stapleton, 2011); second, to be able to acquire critical thinking skills and transfer them beyond high school, students should be taught critical thinking explicitly (Marin & Halpern, 2011). Hence having a shared vision of teaching for critical thinking and realizing that goal requires sufficient knowledge in critical thinking and its instructional strategies and explicit instruction.

Principals', HODs', and teachers' knowledge of critical thinking. The principals and heads of mathematics departments have some idea about critical thinking, but their knowledge is narrow in scope and ignores the affective domain of critical thinking. Principals' and heads of mathematics departments' definition of critical thinking were not comprehensive; they included one or two components of critical thinking like analysis and decision making based on criteria while disregarding other components completely.

Teachers associated critical thinking with higher order thinking, analysis, evaluation, and making reasoned decisions. Teachers often used the terms logical thinking and higher order thinking to replace critical thinking, thus, confusing them with critical thinking. Moreover, some participating teachers had faulty views about critical thinking believing that it is predetermined, an innate ability, or a gift, and its absence limited the teachers' efforts to teach for critical thinking. These results are broadly in line with those of researchers such as Alazzi (2008), Howe (2004), Innabi and El Sheikh (2007), Stapleton (2011), and Yacoubian (2016).

It is worthwhile to mention that unlike previous research conducted by Innabi and El Sheikh (2007) in Jordan, the teachers believed that critical thinking is a continuous process, not bounded by time or subject matter and can be transferred to students' life outside of school. They did not relate critical thinking to crisis or emergencies, but considered it a way of thinking, a way of life. During the conducted interviews, it became apparent that the participants have all heard of critical thinking and it was a part of their vocabulary, but in

reality, they have rarely discussed it with other teachers or during staff meetings. This finding is consistent with previous research conducted by Howe (2004). Howe (2004) stated that critical thinking was a part of the teachers' philosophy of education, but they rarely discussed or thought about it.

When defining critical thinking none of the principals, heads of mathematics departments, or teachers referred to either cognitive skills or dispositions of critical thinking specific to mathematics. Teachers, when discussing using critical thinking in mathematics, focused on problem solving and mainly on two types of problems: geometric proofs and authentic problems. Stapleton (2011) stated that problem solving in mathematics required essential components of critical thinking like reasoning, analysis, and providing evidence.

Teachers' knowledge of instructional strategies that foster critical thinking. Teachers' knowledge of instructional strategies that enhance students' critical thinking was limited, narrow in scope, and included some discrepancies; these results can be explained by the lack of critical thinking courses in teacher preparation programs or university requirements; and the absence of professional development and trainings about teaching critical thinking. In general, instructional techniques such as individual dialogue, research projects, group projects, and class presentations foster students' critical thinking (Abrami et al., 2015; Halpern, 2001; Terenzini et al, 1999; Tsui, 1999).

Moreover, previous research in mathematics education has established that teaching mathematics using active learning strategies like problem-based learning (Firdaus et al. , 2015), mathematical investigation method (Sumarna et al., 2017), or dialogue between students (Aizikovitsh-Udi & Cheng, 2015), developed and improved students' critical thinking. The most common instructional strategy that the teachers claimed to use to develop students' critical thinking was asking leading questions to find the solution of a problem. Nevertheless, classroom observations and the semi-structured interviews established that

teachers usually asked short, convergent questions that were mostly based on recall of properties and procedures that were previously used. Few participants mentioned open-ended questions, but even they stated that time restrictions prevented them from employing this strategy. Mathematics like all other disciplines includes open-ended questions that experts can argue about and addressing these questions in the classroom encourages students to think critically instead of being passive recipients of knowledge (Browne & Freeman, 2000).

Moreover, all the teachers realized the importance of including real life, authentic problems to develop students' critical thinking and motivate them to do mathematics. This approach is corroborated by previous research conducted by Aizikovitsh-Udi and Cheng (2015) that indicated that applying mathematics to real life problems fosters critical thinking. However, when the teachers were asked to provide examples of authentic problems they have solved in their classrooms, they either failed to do so or provided examples which do not satisfy the requirements of an authentic problem. It can be concluded that teachers' knowledge and classroom application of instructional strategies that enhance critical thinking is limited and not comprehensive. Teachers' beliefs and knowledge influence their instructional practices (Bray, 2011), and the determination of instructional strategies that enhance critical thinking is helpful to teachers in realizing the goal of fostering students' critical thinking (Tsui, 1999).

Moreover, the four schools have not organized any professional development or trainings about critical thinking; hence, the teachers' narrow scope of understanding and misconceptions about critical thinking and how to teach critical thinking. This is not in accord with the role of school leaders in shaping the school culture by establishing a shared school vision and being committed to accomplishing its goals. In a collaborative school culture, the faculty is aware and committed to a shared vision or purpose of the school (Gruenert, 2005; Hargreaves, 1995). It can be concluded that the principals' claim that their

school mission fosters critical thinking is in accordance with the conclusions in previous research conducted by Howe (2004) that school leaders often use critical thinking as an educational catchword without neither discussing it deeply nor trying hard to understand its implications on education.

Most of the principals and heads of mathematics departments have not received formal education on critical thinking. Similarly, most teachers stated that they have not received formal education on critical thinking, neither during their university years nor as professional development. The remaining teachers claimed that they were exposed to critical thinking in their graduate studies at the university. An emerging theme unique to the Lebanese context was that teachers who have continued their graduate studies and graduated in recent years were more cognizant about critical thinking and more motivated to teach for critical thinking compared to their counterparts of teachers with more than 15 years of experience. It is worthwhile to mention that the graduate studies of these teachers are in the domains of educational leadership and mathematics education. This is corroborated by the views of a principal who stated that teachers who have graduated from universities during the last ten years had a better understanding of critical thinking. Classroom observations of these three teachers showed that they solved problems through class discussions led by the teacher where the students tried to find the answers with the teacher occasionally guiding them through questions.

Limitations of the Study

The research was conducted on a small number of private schools; to gain a broader understanding of the problem the research can be conducted on public schools as well. For the purpose of the study, only mathematics teachers' views were considered, similar research conducted on science and social studies teachers can add insight and investigate the similarities and differences between the views of teachers of different subjects. Moreover,

the views of the students about their self-reported improvement of critical thinking skills can also add to the research.

Another limitation is that each classroom was observed only once and during different parts of the lesson; introducing the lesson, solving problems, or reinforcing previous knowledge. Frequent observations during different parts of the lesson would have yielded more information, especially if the teachers were asked to teach for critical thinking. School culture was investigated based on the views of principals and heads of mathematics departments as school leaders that shape the school culture. Future research can also include the views of the teachers and students about their school culture as well as quantitative data about the school culture and students' critical thinking abilities.

This study was conducted within the context of mathematics education and it was not within the scope of this study to delve into the epistemology of mathematics and how it is related to critical thinking and school culture. Future research should probe into the relation between the epistemology of mathematics and creating a school culture conducive to critical thinking.

Recommendations for Practice

School leaders should initiate discussions with teachers about the epistemological orientation of the school as well as guide and support teachers to align instruction with these views. Wright (2017) demonstrated that encouraging teachers to think about their epistemologies helps them change their instructional strategies to more student-centered and active learning. Moreover, to ensure that teachers are cognizant in the instructional strategies of critical thinking, school leaders should provide training and professional development in teaching for critical thinking. School leaders should create collaborative school cultures based on collaborative leadership and teacher collaboration to empower teachers to initiate change and develop their students' critical thinking.

References

Abolghasemi, M., McCormick, J., & Conners, R. (1999). The importance of department heads in the development of teacher support for school vision. *International Journal of Educational Management, 13*(2), 80-86. doi: 10.1108/09513549910261159

Abrami, P. C., Bernard, R. M., Borokhovski, E., Waddington, D. I., Wade, C. A., & Persson, T. (2015). Strategies for teaching students to think critically: A meta-analysis. *Review of Educational Research, 85*(2), 275-314. doi: 10.3102/0034654314551063.

Aizikovitsh-Udi, E. & Cheng, D. (2015). Developing critical thinking skills from dispositions to abilities: Mathematics education from early childhood to high school. *Creative Education, 6*(4), 455- 462. doi: 10.4236/ce.2015.64045

Alazzi, K. F. (2008). Teachers' perceptions of critical thinking: a study of Jordanian secondary school social studies teachers. *The Social Studies, 99*(6), 243-248. doi: 10.3200/tsss.99.6.243-248.

Aliakbari, M. & Sadeghdaghighi, A. (2013). Teachers' perception of the barriers to critical thinking, *Procedia - Social and Behavioral Sciences, 70*, 1-5. doi: 10.1016/j.sbspro.2013.01.031

American Philosophical Association. (1990). *Critical Thinking: A Statement of Expert Consensus for Purposes of Educational Assessment and Instruction.* "The Delphi Report," Committee on Pre-College Philosophy. (ERIC Doc. No. ED 315 423).

Ampadu, E. (2014). Implementing a new mathematics curriculum: Mathematics teachers' beliefs and practices. *International Journal of Research Studies in Education, 3*(1), 79-90. doi: 10.5861/ijrse.2013.562.

Apple, M. W. (1992). Do the standards go far enough? Power, policy, and practice in mathematics education. *Journal for Research in Mathematics Education, 23*(5), 412-431. Retrieved from https://www.jstor.org/stable/749562. doi: 10.2307/749562

Baxter, P., & Jack, S. (2008). Qualitative case study methodology: Study design and implementation for novice researchers. *The Qualitative Report, 13*(4), 544- 559.

Bray, W. S. (2011). A collective case study of the influence of teachers' beliefs and knowledge on error-handling practices during class discussion of mathematics. *Journal for Research in Mathematics Education, 42*(1), 2-38. doi: 10.5951/jresematheduc.42.1.0002

Brooks, J., & Normore, A. H. (2015). Qualitative research and educational leadership: Essential dynamics to consider when designing and conducting studies. *International Journal of Educational Management, 29,* 798-806.

Browne, M. N., & Freeman, K. (2000) Distinguishing Features of Critical Thinking Classrooms. *Teaching in Higher Education, 5*(3), 301-309. doi: 10.1080/713699143

Busher, H., & Harris, A. (1999). Leadership school subject areas: Tensions and dimensions of managing in the middle. *School Leadership and Management, 9(3),*305-317.

Center for Educational Research and Development (1999). Curricula and Objectives of General Education. Beirut: CERD.

Choy, S. C., & Cheah, P. K. (2009). Teacher perceptions of critical thinking among students and its influence on higher education. *International Journal of Teaching and Learning in Higher Education, 20*(2), 198-206.

Chukwuyenum, A. N. (2013). Impact of critical thinking on performance in mathematics among senior secondary school students in Lagos state. *IOSR Journal of Research and Method in Counseling Education, 3*(5), 18-25. doi: 10.9790/7388-0351825

Creswell, J. W. (2003). *Research design: Qualitative, quantitative and mixed methods approaches* (2nd ed.). Thousand Oaks, CA: Sage Publications

Creswell, J. W., Hanson, W. E., Clark, V. L., & Morales, A. (2007). Qualitative research designs: selection and implementation. *The Counseling Psychologist, 35*(2): 236- 264. doi: 10.1177/0011000006287390

Deal, T. E., & Peterson, K. D. (1990). The principal's role in shaping school culture. Washington, D.C.: U.S. Dept. of Education, Office of Educational Research and Improvement, Education Resources Information Center. Programs for the improvement of practice.

Engels, N., Hotton, G., Devos, G., Bouckenooghe, D., & Aelterman, A. (2008). Principals in schools with a positive school culture. *Educational Studies, 34*(3), 159-174, doi: 10.1080/03055690701811263

Ennis, R. H. (1964). A definition of critical thinking. *The Reading Teacher, 17*(8), 599-612.

Ennis, R. H. (1985). A logical basis for measuring critical thinking skills. *Educational Leadership, 43*(2), 44-48. www.ascd.org/ASCD/pdf/journals/ed_lead/el_198510_ennis.pdf

Ennis, R. H. (2013a). Critical thinking across the curriculum: The Wisdom CTAC Program. *Inquiry: Critical Thinking across the Curriculum, 28*(2), 25-45. doi: 10.5840/inquiryct20132828

Ennis, R. H. (2015). Critical thinking: A streamlined conception (An expanded and revised version of 1991a, below). In M. Davies, & R. Barnett (Eds.), *A handbook of critical thinking in higher education* (31-47). New York: Palgrave Macmillan.

Ernest, P. (1989). The impact of beliefs on the teaching of mathematics. In P. Ernest (Ed.), *Mathematics teaching: The state of the art* (249-254). London: The Falmer Press.

Facione, P. A. (1990). *Critical thinking: a statement of expert consensus for purposes of educational assessment and instruction. Executive summary: the Delphi report*. Millbrae, CA: The California Academic Press

Firdaus, F., Kailani, I., Bakar, M. N. B., & Bakry, B. (2015). Developing critical thinking skills of students in mathematics learning. *Journal of Education and Learning(EduLearn), 9*(3), 226- 236. doi: 10.11591/edulearn.v9i3.1830

Gates, P. (2006). Going beyond belief systems: exploring a model for the social influence on mathematics teacher beliefs. *Educational studies in mathematics, 63*(3), 347-369. doi: 10.1007/s10649-005-9007-z.

Gill, M. G., & Boote, D. N. (2012). Classroom culture, mathematics culture, and the failures of reform. The need for a collective view of culture. *Teachers College Record, 114*(12), 1-45.

Gruenert, S. (2000). Shaping a new school culture. *Contemporary Education, 71*(2), 14-17.

Gruenert, S. (2005). Correlations of collaborative school cultures with student achievement. *National Association of Secondary School Principals (NASSP) Bulletin, 89*(645), 43-55. doi: 10.1177/019263650508964504

Gutiérrez, R. (1996). Practices, beliefs and cultures of high school mathematics departments: understanding their influence on student advancement, *Journal of Curriculum Studies, 28*(5), 495-529, doi: 10.1080/0022027980280501

Hallinger, P. (2003). Leading educational change: Reflections on the practice of instructional and transformational leadership. *Cambridge Journal of Education, 33*(3), 329- 352. doi: 10.1080/0305764032000122005

Hallinger, P., & Heck, R. H. (2010). Collaborative leadership and school improvement: understanding the impact on school capacity and student learning. S*chool Leadership and Management, 30*(2), 95-110. doi: 10.1080/ 13632431003663214.

Halpern, D. F. (1999). Teaching for critical thinking: Helping college students develop the skills and dispositions of a critical thinker. *New Directions for Teaching and Learning, 1999*(80), 69 - 74.

Halpern, D. F. (2001). Assessing the effectiveness of critical thinking instruction. *The Journal of General Education, 50*(4), 270 - 286. doi: 10.1353/jge.2001.0024

Hargreaves, D. H. (1995). School culture, school effectiveness and school improvement. *School Effectiveness and School Improvement, 6*(1), 23-46. doi: 10.1080/0924345950060102

Hargreaves, D. H. (1999), `Helping Practitioners Explore Their School's Culture'. In J. Prosser (Ed.), *School Culture* (48-65). London: Paul Chapman Publishing Ltd.

Heck, R. H., & Marcoulides, G.A. (1996). School culture and performance: testing the invariance of an organizational model. *School Effectiveness and School Improvement, 7*(1), 76-95, doi: 10.1080/0924345960070104.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, *28*(5), 524-549. doi: 10.2307/749690

Henningsen, M.A., & Zebian, S. (2003). High-level thinking, reasoning, and communication in Lebanese elementary mathematics classrooms: A preliminary technical report on the fall 2002-2003 classroom observations of the MARAL project.

Hongboontri, C., & Keawkhong, N. (2014). School Culture: Teachers' Beliefs, Behaviors, and Instructional Practices. *Australian Journal of Teacher Education, 39*(5), 66-88. Retrieved from http://ro.ecu.edu.au/ajte/vol139/iss5/5. doi: 10.14221/ajte.2014v39n5.7

Howe, E.R. (2004). Canadian and Japanese teachers' conceptions of critical thinking: a comparative study, *Teachers and Teaching, 10*(5), 505- 525. doi: 10.1080/1354060042000243051

Innabi, H., & El Sheikh, O. (2007). The change in mathematics teachers' perceptions of critical thinking after 15 years of educational reform in Jordan. *Educational Studies in Mathematics, 64*(1), 45-68.

Jacob, S. M. (2012). Mathematical achievement and critical thinking skills in asynchronous discussion forums. *Procedia- Social and Behavioral Sciences, 31*, 800-804. doi: 10.1016/j.sbspro.2011.12.144

Johnson, R. H., & Hamby, B. (2015). A meta-level approach to the problem of defining 'critical thinking'. *Argumentation, 29*(4), 417–430. doi 10.1007/s10503-015-9356-4

Leithwood, K. (2016). Department-head leadership for school improvement. *Leadership and Policy in Schools, 15*(2), 117- 140. doi: 10.1080/15700763.2015.1044538

Lipman, M. (1988). Critical thinking - What can it be? *Educational Leadership, 46*(1), 38-43.

Magno, C. (2010). The role of metacognitive skills in developing critical thinking. *Metacognition and Learning, 5*(2), 137- 156. doi: 10.1007/s11409-010-9054-4.

Marin, L.M., & Halpern, D.F. (2011). Pedagogy for developing critical thinking in adolescents: Explicit instruction produces greatest gains. *Thinking Skills and Creativity, 6*(1), 1-13. doi: 10.1016/j.tsc.2010.08.002.

Maslowski, R. (2001). *School culture and school performance: An explorative study into the organizational culture of secondary schools and their effects.* Twente: University of Twente Press.

McCartney, C., & Schrag, F. (1990). Departmental and school leadership in promoting higher-order thinking. *Journal of Curriculum Studies, 22*(6), 529-543. doi: 10.1080/0022027900220602

Miles, M. B., Huberman, A. M., & Saldaña, J. (2014). *Qualitative Data Analysis: A Methods Sourcebook* (3rd Ed.). Thousand Oaks, CA: Sage.

Ministry of Education. (2006). *Education strategies in Lebanon*. Beirut: Lebanon.

Nabhani, M., Busher, H., & Bahous, R. (2012). Cultures of engagement in challenging circumstances: four Lebanese primary schools in urban Beirut. *School Leadership and Management, 32*(1), 37-55. doi: 10.1080/13632434.2011.642353.

Nardi, E., & Steward. S. (2003). Is Mathematics T.I.R.E.D.? A Profile of Quiet Disaffection in the Secondary Mathematics Classroom. *British Educational Research Journal, 29*(3), 345–366. doi: 10.1080/01411920301852

Nelson, B. S., & Sassi, A. (2000). Shifting approaches to supervision: the case of mathematics Supervision. *Educational Administration Quarterly, 6*(4), 553- 584. doi: 10.1177/00131610021969100

NCTM. (1989). Principles and standards for school mathematics. Reston, VA: Author.

Pajares, M. F. (1992). Teachers' beliefs and educational research: cleaning up a messy construct. *Review of Educational Research, 62*(3), 307- 332. doi: 10.2307/1170741

Paul, R. W. (1985). Bloom's Taxonomy and Critical Thinking Instruction. *Educational Leadership, 42*(8), 36-39.

Paul, R. (2005). The state of critical thinking today. *New Directions for Community Colleges, 2005*(130), 27-38. doi: 10.1002/cc.193

Prosser, J. (1999). School Culture. London: SAGE Publications.

Radulovic, L. & Stancic, M. (2017). What is needed to develop critical thinking in schools? *CEPS Journal, 7*(3), 9-25.

Schein, E. (1992). *Organizational culture and leadership,* San Francisco: Jossey-Bass.

Siegel, H. (1985). Educating reason: Critical thinking, informal logic, and the philosophy of education. *Informal Logic, 7*(2), 69-81. doi: http: // dx.doi.org/ 10.22329/il.v7i2.2706

Siskin,L.S. (1991). Departments as different worlds: subject subcultures in secondary schools. *Educational Administration Quarterly*, 27(2), 134-160.

Smith, V. G., & Szymanski, A. (2013). Critical thinking: more than test scores. *International Journal of Educational Leadership Preparation, 8*(2), 16-25.

Stapleton, P. (2011). A survey of attitudes towards critical thinking among Hong Kong secondary school teachers: Implications for policy change. *Thinking Skills and Creativity, 6*(1), 14-23 doi: 10.1016/j.tsc.2010.11.002

Su, H. F., Ricci, F.A., & Mnatsakanian, M. (2016). Mathematical teaching strategies: Pathways to critical thinking and metacognition. *International Journal of Research in Education and Science (IJRES), 2*(1), 190-200. doi: 10.21890/ijres.57796

Sumarna, N., Wahyudin, & Herman, T. (2017). The increase of critical thinking skills through mathematical investigation approach. *Journal of Physics: Conference Series, 812.* doi: 10.1088/1742-6596/812/1/012067

Terenzini, P. T., Pascarella, E. T., & Blimling, G. S. (1999). Students' Out-of-Class Experiences and their Influence on learning and cognitive development: A literature review. *Journal of College Student Development, 40*(5), 610-623. Retrieved from https://eric.ed.gov/?id=EJ614286

Terenzini, P. T., Springer, L., Pascarella, E. T., & Nora, A. (1995). Influences affecting the development of students' critical thinking skills. *Research in Higher Education, 36*(1), 23-39. doi: 10.1007/bf02207765

Tsui, L. (1999). Courses and instruction affecting critical thinking. *Research in Higher Education, 40(2),* 185-200.

Tsui, L. (2000). Effects of Campus Culture on Students' Critical thinking. *The Review of Higher Education, 23*(4), 421- 441. doi: 10.1353/ rhe.2000.0020.

Tsui, L. (2002). Fostering critical thinking trough effective pedagogy: Evidence from four institutional case studies. *The Journal of Higher Education, 73*(6), 740-763. doi: 10.1353/jhe.2002.0056

White, W. F. & Hargrove, R. (1996). Are those preparing to teach prepared to teach critical thinking? *Journal of Instructional Psychology, 23,* 117-120.

Wright, P. (2017). Critical relationships between teachers and learners of school mathematics. *Pedagogy, Culture & Society*, *25*(4), 515-530. doi:10.1080/14681366.2017.1285345

Yacoubian, H. A. (2016). *Critical Thinking at the Armenian Schools in Lebanon* [in Armenian]. Beirut: Haigazian University Press.

Yin, R. K. (2003). *Case study research: Design and methods* (3rd ed.). Thousand Oaks, CA: Sage.

Zakaria, E., & Maat, S. M. (2012). Mathematics teachers' beliefs and teaching practices. *Journal of Mathematics and Statistics, 8*(2), 191-194.

APPENDIX A

Initial Contact Letter

Study Title: Critical Thinking, Mathematics Classroom, and School Culture: Views of Principals, Heads of Mathematics Departments, and Secondary Mathematics Teachers in Lebanon

Dear Date:

In partial fulfillment of the requirements for the degree of Master of Arts in Education at Haigazian University, I would like to ask you to participate in a research study that I am conducting, entitled "Critical Thinking, Mathematics Classroom, and School Culture: Views of Principals, Heads of Mathematics Departments, and Secondary Mathematics Teachers in Lebanon". Insights gained from this research will be beneficial for school leaders and teachers in Lebanon to create a school culture that fosters and develops their students' critical thinking, which is considered one of the most important skills of the 21st century.
The purpose of this research is (1) to study the views of principals, heads of departments, and mathematics teachers in secondary schools in Lebanon on the role of critical thinking in mathematics, and (2) to investigate school leaders' and heads of departments' views of their role in creating a school culture conducive to critical thinking.

You are expected to fill a questionnaire and participate in an interview. The duration of the interview is about 60 minutes and it will be recorded. I will also observe mathematics classrooms.

Kindly note that your participation in the study is completely voluntary and you are not obliged to answer any specific questions even if you had given initial consent of participation. You can withdraw from the study at any time, for any reason, without penalty.

I also ensure you that all data collected will be used only for academic purposes and will be treated confidentially. Your name, school name and contact information will not be used in the final report. Only the researcher will have access to the transcripts and questionnaires. If you have any further questions about the study, please do not hesitate to contact me at XX-XXXXXX or e-mail at XX-XXXXXX. You may also contact my thesis advisor, Dr Hagop Yacoubian, at XXXXXXXXXXX.

I deeply appreciate your participation in this study.

Respectfully,

Taline Hawatian

MA candidate

APPENDIX B

Questionnaire for Teachers

Name and Surname: Gender:

School name:

Teacher's phone number:

Teacher's email:

Grade level teaching:

Number of years teaching:

Number of years teaching in your current school:

Educational level/ University name:

Area of Specialization:

Area I: Critical thinking in mathematics

1. In your opinion, what is critical thinking and what does it include?
2. In your opinion, do your students need your guidance to develop their critical thinking? Justify your answer.
3. How much do you incorporate critical thinking in your everyday mathematics teaching? Justify your answer.

Area II: The role and importance of critical thinking in learning mathematics.

1. As a mathematics teacher, do you think that you have a role to enhance critical thinking of your students? Justify your answer.
2. In your opinion, how much does the mathematics curriculum that you use foster the development of critical thinking of your students? Please justify your answer.
3. Do you think that teaching for critical thinking would help students learn mathematics in a better way? How? Please explain by giving examples.

4. In your opinion, to what extent it is possible to develop students' critical thinking while teaching mathematics? Justify your answer.

Area III: Teaching strategies that could help students improve their critical thinking

1. What strategies and techniques do you use in teaching mathematics to help your students improve their critical thinking? Give examples.
2. How can you evaluate that a student is thinking critically when solving mathematics problems? Explain.
3. Think about a classroom activity that you recently planned and carried out and through which it became possible to develop critical thinking among your students. Please describe the details of this activity, mention your role, and how it fostered the development of students' critical thinking.

Area IV.- Barriers to teaching critical thinking in mathematics

1. What are some of the challenges that you face when you want to foster the development of students' critical thinking?
2. How have you acquired your pedagogical content knowledge in critical thinking?
3. In your professional development, have you been provided with opportunities to increase your pedagogical content knowledge of critical thinking? Please explain.
4. Suggestions and comments.

This questionnaire is prepared based on the literature review and the questions formulated by Innabi & El Sheikh (2007) and Yacoubian (2016)

APPENDIX C

Questionnaire for Heads of departments

Name and Surname: Gender:

School name:

HOD's phone number:

HOD's email

Experience:

Years of experience as HOD in your current school:

Educational level/ University name:

Area of Specialization:

1. In your opinion, what is critical thinking and what does it include?
2. In your opinion, how do your school mission and goals foster students' critical thinking?
3. In your opinion, and as an HOD of mathematics, do you have a role to enhance critical thinking of your students? Justify your answer.
4. As an HOD, do you support innovation and risk taking among teachers? If yes, please describe an instance when a teacher used a new instructional strategy. How did you support or reward the teacher for experimenting with new ideas and techniques? Please elaborate.
5. Think of an important decision related to school policy that you were involved in making recently. Please describe how you participated in decision making and the steps you took to implement it.
6. How much time do teachers allocate for dialogue between peers, planning across grades and subjects, and working together to evaluate programs? Please elaborate.

7. How are disagreements about teaching practices resolved? Please give examples if possible.
8. How would you describe the relationship of your teachers with each other? Do they trust and value each other?
9. Describe an instance in your department when teachers worked cooperatively to solve a problem. Please explain the steps they took to find a solution and how they solved the problem.
10. How have the teachers in the mathematics department developed their pedagogical content knowledge in critical thinking? Please mention workshops or seminars they have participated in or the school has organized.
11. What role do you play in creating professional development opportunities for the teachers?
12. What is the dominant mode of instruction in the mathematics classroom? How do students acquire knowledge? How do teachers engage students in learning?
13. Think of a classroom activity that you recently observed and that engaged students in active learning. Please describe this activity in detail, setting of the classroom, the role of the teachers and the students.
14. In your opinion, how do mathematics teachers in your school engage students in critical thinking while teaching mathematics? Please elaborate.
15. In your school are students encouraged to engage in debates about controversial political issues? If yes, please provide an example. If no, explain why not.
16. In your school are students encouraged to engage in debates about controversial social issues? If yes, please provide an example. If no, please explain why not.
17. In your opinion, what are the factors that affect students' critical thinking? Please elaborate.

18. As an HOD, how do you encourage teachers to teach critical thinking in the mathematics classroom?
19. In your opinion, what aspects of your school culture inhibit teachers' efforts to teach critical thinking in the mathematics classroom?

APPENDIX D

Questionnaire for Principals

Name and Surname: Gender:

School name:

Principal's phone number:

Principal's email

Administrative Experience:

Educational level/ University name:

Area of Specialization:

1. In your opinion, what is critical thinking and what does it include?
2. In your opinion, how do your school mission and goals foster students' critical thinking?
3. In your opinion, as a principal do you have a role to enhance critical thinking of your students? Please elaborate.
4. What aspects of your school culture encourage students to think critically?
5. As a principal, do you support innovation and risk taking in teaching? If yes, please describe an instance when a teacher used a new instructional strategy. How did you support or reward the teacher for experimenting with new ideas and techniques? Please elaborate.
6. Think of an important decision related to school policy that you made recently. Please describe how you made that decision and the steps you took to implement it.
7. How much time do teachers allocate for dialogue between peers, planning across grades and subjects, and working together to evaluate programs? Please elaborate.

8. How are disagreements about teaching practices resolved? Please give examples if possible.
9. How would you describe the relationship of your teachers with each other? Do they trust and value each other?
10. Describe an instance in your school when teachers worked cooperatively to solve a problem. Please explain the steps they took to find a solution and how they solved the problem.
11. How have the teachers developed their pedagogical content knowledge in critical thinking? Please mention workshops or seminars they have participated in or the school has organized.
12. What role do you play in creating professional development opportunities for the teachers?
13. On what basis do you choose topics for professional development? Please list the topics of the professional development sessions your teachers have recently attended (the last three years if possible).
14. What is the dominant mode of instruction in your school? How do students acquire knowledge? How do teachers engage students in learning?
15. Think of a classroom activity that you recently observed and that engaged students in active learning. Please describe this activity in detail, setting of the classroom, the role of the teachers and the students.
16. In your opinion, how do mathematics teachers in your school engage students in critical thinking while teaching mathematics? Please elaborate.
17. How do students in your school participate in school life?

18. Describe an experience when as a principal you involved students in school decision making. How did you engage students in the decision making process? Were all the students involved? Did their input affect the final decision? Please elaborate.
19. As a principal, do you encourage students and teachers to engage in debates about controversial political issues? Provide an example of how you guide them.
20. As a principal, do you encourage students and teachers to engage in debates about controversial social issues? Provide an example of how you guide them.
21. In your opinion, what are the factors that affect students' critical thinking? Please elaborate.
22. As a principal, how do you encourage teachers to teach students to think critically?
23. In your opinion, what aspects of your school culture inhibit teachers' efforts to teach critical thinking in general and in particular in the mathematics classroom?